Using a comparison test determine which of the following series converge or diverge. Indicate which test you used and what you concluded from that test.

1. \( \sum_{n=1}^{\infty} \frac{1}{2n^3 + n - 1} \)

2. \( \sum_{n=1}^{\infty} \frac{n^4 + 1}{n^5 + 1} \)

3. \( \sum_{n=2}^{\infty} \frac{\sin n}{n^3 + 13} \)

4. \( \sum_{n=3}^{\infty} \frac{\sqrt{n}}{n^3 - 2} \)

Determine if the following series are absolutely convergent, conditionally convergent, or divergent. Indicate which test you used and what you concluded from that test.

9. \( \sum_{n=1}^{\infty} \frac{(-1)^n}{n^4 + 3n + 1} \)

10. \( \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n} + 1} \)

11. \( \sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln(n)^3} \)

12. \( \sum_{n=1}^{\infty} (-1)^n 2n^2 \)

13. \( \sum_{n=1}^{\infty} \frac{(-1)^n \ln(n)^2}{n} \)

14. \( \sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{n + 1} \)

15. \( \sum_{n=1}^{\infty} \frac{(-1)^n n + 3}{n^3 + 3n^2 - n + 3} \)

16. \( \sum_{n=1}^{\infty} (-1)^n n^2 e^{-2n} \)
Alternating Series Error Estimation Theorem

\[ |R_n| = \left| \sum_{k=1}^{\infty} a_k - \sum_{k=1}^{n} a_k \right| \leq a_{n+1} \]

17. Consider the series \( \sin(1) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} = 1 - \frac{1}{3!} + \frac{1}{5!} - \frac{1}{7!} + \ldots \). (a) What is the alternating series error estimate if the first four terms are used?

\[ |\sin(1) - (1 - \frac{1}{3!} + \frac{1}{5!} - \frac{1}{7!})| < \]

(b) What is the actual difference if the first four terms are used?

\[ |\sin(1) - (1 - \frac{1}{3!} + \frac{1}{5!} - \frac{1}{7!})| = \]

18. Consider the series \( \ln(2) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots \). (a) What is the alternating series error estimate if the first ten terms are used?

(b) What is the alternating series error estimate if the first twenty terms are used?

(c) How many terms are necessary to have an estimate accurate to three decimal places? Find the first error estimate that is less than 0.0005.

19. Consider the series \( \frac{1}{e} = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} = 1 - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!} + \ldots \). (a) What is the alternating series error estimate if the first four terms are used?

(b) What is the alternating series error estimate if the first twenty terms are used? Do not write this in decimal form.

(c) How many terms are necessary to have an estimate accurate to three decimal places? Find the first error estimate that is less than 0.0005.