EXPONENTIAL FUNCTIONS

SECTION 1.3  7-11, 16, 19, 20, 22, 28, 30

7. Since
\[ \frac{g(1)}{g(0)} = \frac{g(2)}{g(1)} = \frac{g(3)}{g(2)} = \frac{g(4)}{g(3)} = 2; \]
the ratios of terms one unit apart are the same, so this appears to be an example of exponential growth. A possible formula is \( g(t) = 2^t \).

8. The ratios of succeeding values of \( f(t) \) are all different:
\[ \frac{f(3)}{f(2)} = 2; \frac{f(4)}{f(3)} = 4.5 \text{ and } \frac{f(5)}{f(4)} = 3; \]
Thus, this is neither exponential growth nor exponential decay.

9. We see that all ratios are the same:
\[ \frac{h(5)}{h(4)} = \frac{h(6)}{h(5)} = \frac{h(7)}{h(6)} = \frac{1}{2} \]
The function is decreasing as \( t \) increases, so this appears to be an example of exponential decay. To find a formula for \( h(t) \), we let \( h(t) = a \cdot b^t \); with \( b = 1/2 \).

Now, we solve for \( a \): \[ 2096 = a \cdot (1/2)^3 \quad a = 2096 / (1/2)^3 = 16768 \]
Thus, a possible formula for \( h(t) \) is \( h(t) = 16768 \cdot (1/2)^t \).

10. One possible graph is shown below.

Note that any increasing and concave down graph would satisfy the stated conditions.
In this case any concave up increasing function will represent the situation.

16. Since \(2^t \to +\infty\) and \(2^{-t} \to 0\) as \(t \to \infty\), this function has no horizontal asymptote as \(t \to \infty\). Since \(2^t \to 0\) and \(2^{-t} \to +\infty\) as \(t \to -\infty\), this function has no horizontal asymptote as \(t \to -\infty\).

19. (a). Let \(Q = Q_0a^t\). Then \(Q_0a^{0.02} = 25.02\) and \(Q_0a^{0.05} = 25.06\). So,

\[
\frac{Q_0a^{0.05}}{Q_0a^{0.02}} = \frac{25.06}{25.02} = 1.001 = a^{0.03} \text{ so } a = 1.001^{\frac{100}{3}} = 1.05
\]

(b) Since \(a = 1.05\), the growth rate is \(r = 0.05 = 5\%\).

20. You can find this equation in two ways:

a) Since the graph looks like an exponential function, and two points are given (0,3), (2,12) we can find the ratio between consecutive values of \(y\), taking into consideration the time interval to find the constant of growth. In this case we have: \(\frac{12}{3} = 4\) in a period of 2 years with an initial population of 3.

Hence, the formula is: \(P(t) = 3 \times 4^\frac{t}{2}\)

b) Algebraically, substitute each of the given points into the formula \(P(x) = Qa^x\). You will find the system of equations:

\[
\begin{cases}
3 = Qa^0 \\
12 = Qa^2
\end{cases}
\]

The solution will produce the same answer as in part (a).
22. You can solve this problem in a similar form as problem 20.
   a) The decay factor for a period of 2 is \( \frac{2}{8} = 0.25 \). So far the formula looks like \( P(t) = P_0 (0.25)^{t/2} \). To put this formula in the form \( P(t) = P_0 a^t \), we notice that \( a = 0.25^{1/2} = 0.5 \). This is to be the decay factor per year. Since we know the population after the first year, we can easily find the initial population by solving \( 1 - 0.5P_0 = 2 \), \( P_0 = 4 \). Therefore, the formula for this is:
   \[
P(t) = 4(0.5)^t
   \]
   b) On the other hand you can solve this algebraically, by solving the system
   \[
   \begin{align*}
   2 &= P_0 a \\
   8 &= P_0 a^{-1}
   \end{align*}
   \]

28. Since the half-life of the strontium-90 is 29 years, we use this information to find \( a \), in the formula \( P(t) = P_0 a^t \). To this end, solve the equation
   \[
   \frac{1}{2} P_0 = P_0 a^{29}, \text{ so } a = \left( \frac{1}{2} \right)^{29}
   \]
   The amount present after 30 years is \( P(30) \), it is \( P_0 \left( \frac{1}{2} \right)^{30} = 0.488 P_0 \)

30. (a) There are 3 years from the end of 1996 till the start of 2000, so
   Population at start of 2000 = 266.5(1.009)^3 = 273.8 million.
   
   (b) There are 7 years from the start of 1990 till the end of 1996, so writing \( r \) for the annual growth rate, we have
   \[
   248.7(1 + r)^7 = 266.5
   \]
   so
   \[
   1 + r = (266.5/248.7)^{1/7} = 1.009924
   \]
   giving \( r = 0.009924 = 0.9924\% \)

32. Two points are given: (0,50) and (20, 100) where \( t=0 \) represents the year 1970, and the price is in units of $1000.
   a. The linear equation that fits these two data points is \( y = 2.5x + 50 \)
   b. The exponential function that fits these two data points is: \( y = 50 \times 2^{x/20} \)