SOLUTIONS TO 5.3: 2, 4, 6, 12, 22, 26

2. The global maximum is achieved at the two local maxima, which are at the same height.

4. (a) \( f'(x) = 1 - \frac{1}{x} \). This is zero only when \( x = 1 \). Now \( f'(x) \) is positive when \( x > 1 \), and negative when \( x < 1 \). Thus \( f(1) = 1 \) is a local minimum. The endpoints \( f(0.1) \approx 2.4026 \) and \( f(2) \approx 1.3069 \) are local maximums.

(b) Comparing values of \( f \) shows that \( x = 0.1 \) gives the global maximum and \( x = 1 \) gives the global minimum.

6. Since the function is positive, the graph lies above the x-axis. If there is a global maximum at \( x = 3 \), \( t'(x) \) must be positive, then negative. Since \( t'(x) \) and \( t''(x) \) have the same sign for \( x < 3 \), they must both be positive, and thus the graph must be increasing and concave up. Since \( t'(x) \) and \( t''(x) \) have opposite signs for \( x > 3 \) and \( t'(x) \) is negative, \( t''(x) \) must again be positive and the graph must be decreasing and concave up. A possible sketch of \( y = t(x) \) is shown in the figure below.
12. We look for critical points of $M$. \( \frac{dM}{dx} = \frac{1}{2} wL - wx \)

Now \( \frac{dM}{dx} = 0 \) when \( x = L/2 \). At this point \( \frac{d^2M}{dx^2} = -w \) so this point is a local maximum. The graph of $M(x)$ is a parabola opening downwards, so the local maximum is also the global maximum.

22. The graph of $y = x + \sin x$ below suggests that the function is non-decreasing over the entire interval. You can confirm this by looking at the derivative $y' = 1 + \cos x \geq 0$.

26. (a) To obtain $g(v)$, which is in gallons per mile, we need to divide $f(v)$ (in gallons per hour) by $v$ (in miles per hour). Thus, $g(v) = \frac{f(v)}{v}$.

(b) By inspecting the graph, we see that $f(v)$ is minimized at approximately 220 mph.

(c) Note that a point on the graph of $f(v)$ has the coordinates $(v; f(v))$. The line passing through this point and the origin $(0, 0)$ has

$$\text{Slope} = \frac{f(v) - 0}{v - 0} = \frac{f(v)}{v} = g(v)$$

So minimizing $g(v)$ corresponds to finding the line of minimum slope from the family of lines which pass through the origin $(0, 0)$ and the point $(v, f(v))$ on the graph of $f(v)$. This line is the unique member of the family which is tangent to the graph of $f(v)$. The value of $v$ corresponding to the point of tangency will minimize $g(v)$. This value of $v$ will satisfy $f(v) = v = f'(v)$. From the graph below, we see that $v \approx 300$ mph.