Some comments about the use of universal and existential quantifiers.

We will consider two propositions to clarify the use of existential and universal quantifiers with connectives. For our case the universe is the set of all the fruits. Let
\[ A(x): x \text{ is an apple}, \]
\[ S(x): x \text{ has spots}. \]

For the proposition “All apples have spots”, we have two possibilities to write the statement symbolically:

a. \( (\forall x)(A(x) \land S(x)) \) or
b. \( (\forall x)(A(x) \rightarrow S(x)) \)

The first statement means that all the fruits are apples with spots. This is not the meaning of our statement.
For the second one, it says that if a fruit is an apple, it has spots. This is the intended meaning.

For the proposition “Some apples have spots” we can write it in symbols as

a. \( (\exists x)(A(x) \land S(x)) \) or
b. \( (\exists x)(A(x) \rightarrow S(x)) \)

The first proposition indicates that some of the fruits satisfy the conditions of being apples and to have spots, which is the intended meaning of our statement.
The second propositions says that for some fruits if they apples they have spots.
However, observe that it does not guarantee the existence of apples with spots. This statement is true for any fruit which is not an apple.

Hence, statements of the form

“All \( P(x) \) are \( Q(x) \)” is symbolized as \( (\forall x)(P(x) \rightarrow Q(x)) \)

and statements of the form “Some \( P(x) \) are \( Q(x) \)” are symbolized as \( (\exists x)(P(x) \land Q(x)) \)