COMMENTS ON STRINGS, PERMUTATIONS, PERMUTATIONS WITH
REPETITIONS, COMBINATIONS AND COMBINATIONS WITH
REPETITIONS.

Permutations on a set with k elements
These can be considered as strings of length k with k different characters. Use the
multiplication principle to count them. There are k! of them.
In the case of r-permutations, these are strings with r different characters, where the r
characters are chosen out of k. There are \( \binom{k}{r} r! \) of them.

Combinations
When we think about combinations we think about subsets. These are some examples of
situations that can be interpreted as subsets (although some of them can be interpreted
using permutations)

To play the Texas lotto six numbers are chosen out of 49. Since order does not matter we
can think of each number as a subset with six elements from a set with 49 elements.

How many 8-bit strings are there with 3 zeroes? We learned that this problem can be
solved using permutations with repetitions. However, it can also be solved using
combinations. This is the way to interpret it so that it can be solved using combinations.
Label the slots from 1 through 8. Since there are 3 zeroes, we need to choose three slots
to place the zeroes, and then from the remaining ones choose five to place the five ones,
or what is the same choose a subset of three elements from the 8-set. This can be done in
\( \binom{8}{3} \) different ways.

Permutations with repetitions

How many words can we make with 3 R’s, 2 B’s and 4 C’s?
In this case we are dealing with strings of length 9. Since the order matters and there are
repeating characters, it is a permutation with repetitions.
To do the counting, we can approach the situation as a combination problem in a similar
manner as we solve the bit string problem. The total number of such strings will be
\[
\binom{9}{3} \binom{9 - 3}{2} \binom{9 - 3 - 2}{4} = \frac{9!}{3! 2! 4!}
\]

How many 8-bit strings are there with at most 4 zeroes? This problem involves the IEP
and permutations with repetitions.

Combinations with repetitions
There are five students, call them 1, 2, 3, 4, 5, amongst who we will distribute three different types of cookies, named A, B, C. In how many ways can we pass the cookies out if we do not care who gets what, and each student will get for sure one cookie? This should be interpreted as follows: three A cookies can be passed out to 1, 2, 3, or 1, 2, 4, or, 1, 2, 5, or, ..., 3, 4, 5. In this case, it counts just as one way of distributing three A cookies.

Let’s see why we cannot set this problem up as a string problem from the very beginning. To approach this problem as a string problem the only option we have is to consider 5-strings, with slots labeled 1, 2, …, 5, due to the fact that to each student corresponds a cookie. These are some strings indicating how to assign three A cookies, and two C cookies: AAACC, ACCAA, CAACA, etc. All these count as one-way to distribute three A cookies and two C cookies. Since we care about assigning the types of cookies, let’s label them as follow

\[
\begin{array}{ccc}
A & / & B & / & C \\
*** & / & / & **
\end{array}
\]

The stars represent the students and we do not care about the names of the kids getting the same cookie. We just care about how many cookies of that type are distributed. These are other ways to give the cookies away,

\[
\begin{array}{ccc}
A & / & B & / & C \\
*** & / & / & **
* & / & *** & / & *
/ & ***** & / \\
*** & / & ** & /
\end{array}
\]

Observe that in each case we have a string formed by five *s (students), and two /s (types of cookies minus 1). So, counting the total number of ways of distributing the cookies is the same as counting all possible strings we can make with five *s and two /s.

**EXERCISE:**
Determine whether each of the problems that follow correspond to a combination with repetitions.
Six warehouses is each to receive one shipment of paint, hammers, or shingles. In how many ways can this be done?

How many bit strings are there with three 0’s and two 1’s?

One bingo card is to be distributed to each of 12 players. In how many ways can this be done, if there are 15 kinds of cards and repetitions are allowed?

Include one with bit strings.