PERMUTATIONS AND COMBINATIONS

1. Identify when a situation corresponds to a permutation, or a combination.

   For each of the situations below indicate whether it corresponds to a permutation or a combination

1.1. How many different 4-digit numbers can you make using all the numbers 2, 5, 7, 9?

1.2. How many different numbers are there in Texas lottery?

1.3. How many ways are there to arrange the letters a, b, c and d such that a is not followed immediately by b?

1.4. How many ways are there to seat six people around a circular table, where seating is considered to be the same as another if they can be obtained from each other by rotating the table?

1.5. How many ways are there to seat six people around a circular table, where seating is considered to be the same if they can be obtained from each other by rotating the table?

1.6. How many strings of length five can we make with the characters M, m, 2, 3, 0 such that the third character is always a letter and the last one a number? The other characters can be either a letter or a number. Give the answer in the form 3*4*5

1.7. There are fifteen players in a soccer team. In how many ways can we place eleven players on the field?

1.8. From a standard 52-card deck you want to obtain a 7-card hand having for sure two aces and three tens. In how many ways can this happen?

2. Interpret how to count number of combinations as number of subsets of a certain set.

   2.1. How many subsets with three elements does a set with fifteen elements have?

   2.2. Double-check that \( \binom{10}{3} = \binom{10}{7}, \binom{7}{3} = \binom{7}{4} \). Why are these numbers the same?

   Look at a set and its complement.

   2.3. We know that the number of elements of the power set of a set with n elements is \( 2^n \). Using the interpretation of \( \binom{n}{k} \) to count subsets of size k, explain why the following equation is true.
3. Solve problems using combinations and/or permutations.

3.1. How many bits of length 10:
   3.1.1. Begin with exactly three ones and end with 10?
   3.1.2. Have exactly three zeroes?
   3.1.3. Have the same number of zeroes as ones?
   3.1.4. Have at least seven ones?
   3.1.5. Have four zeroes and two zeroes are not consecutive?
3.2. How many different words of length four can you make with the letters in the word AMIGO?
3.3. How many permutations of the ten digits begin with the three digits 987, contain the digits 45 in the fifth and sixth positions, or end with the three digits 123?

4. Pigeon Hole Principle

4.1. What is the minimum number of students in a class to guarantee that at least three have a last name starting with the same letter?
4.2. Show that if seven integers are selected from the set \{9,10,11,\ldots,18\}, there must be at least two pairs of these integers that their sum is 27.
   Is the conclusion in part (a) true if six integers are selected instead of seven?
4.3. A computer network consists of six computers. Each computer is directly connected to at least one of the other computers. Show that there are at least two computers in the network that are directly connected to the same number of other computers.

5. Identify when a situation corresponds to a permutation, permutation with repetitions, combination, or combination with repetitions, and explain why that is the case.

5.1. How many different strings reordering the letters of the word SUCCESS can you make?
5.1.1.
5.2. A professor writes 40 distinct true/false questions. Of the statements in these questions, 18 are false. He wants to write the questions such that there are not two consecutive false ANSWERS. How many different answer keys can be made?
5.3. At a birthday party, a mother serves a cookie to each of eight children. There are chocolate chips, peanut butter, and oatmeal cookies. In how many ways can each kid get a cookie if there are only three oatmeal cookies and plenty of the others?

6. Able to count permutations, combinations, permutations with repetitions, and combinations with repetitions.
6.1. There are 10 questions on a discrete mathematics final exam. How many ways are there to assign scores to the problems if the sum of the scores is 100 and each question is worth at least 5 points? (Hint: This can be approached as combinations with repetitions).

6.2. How many permutations of the 26 letters of the English alphabet do not contain any of the strings fish, rat, or bird?

6.3. How many strings of 20 decimal digits are there that contain two 0s, four 1s, three 2s, one 3, two 4s, three 5s, two 7s, and three 9s?

6.4. A book publisher has 3000 copies of the discrete math book. How many ways are there to store them in their three warehouses if the copies of the book are indistinguishable?

6.5. There are four different colored liquids to fill in containers. We need to fill out exactly 24 containers guaranteeing that there at least two of each color. In how many ways can we do it?

7. Set up the solution to problems using combinations and/or permutations along with any of the counting techniques.

7.1. Consider passwords of length 8 made with the character 1, 7, m and z.
   7.1.1. How many are there containing exactly three ones and at most two z’s?
   7.1.2. How many are there, CONTAINING THE FOUR CHARACTERS, such that two consecutive characters are distinct?

7.2. How many do have three 1’s, two m’s, two z’s, and one z?

7.3. Show that if seven integers are selected from the first ten positive integers, there must be at least two pairs of these integers with sum eleven.
   Is the conclusion in part (a) true if six integers are selected instead of seven?

7.4. Prove that if four numbers are chosen from the set \{1,2,3,4,5,6\} at least one pair has to add up to 7. Explain the principles you use.

7.5. Suppose that the population of California is approximately 30 million people. What is the minimum number of people who have the same three initials and were born the same day of the year? (not necessarily the same year!).

7.6. a) In this problem you will count the number of paths between the point (0,0) and the point (5,3), where each path is made of northward or eastward steps. One such a path is illustrated on the graph.
   b) How many of these paths start with two eastward steps?

7.7. There are 51 houses on a street. Each house has an address between 1000 and 1099, inclusive. Show that at least two houses have addresses that are consecutive numbers.

7.8. How many ways are there to seat three married couples around a table if husband and wife do not sit next to each other?