Acceleration of New Iterations for Linear Systems and their Preconditioners.

A Proposal for a Thesis in Mathematics

by

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Abstract

Solving large systems of linear equations is a very important research area due to the many applications that these systems have. The use preconditioning is one of the important techniques for accelerating convergence of the iterative methods.

Tarazaga and Cuellar introduced new diagonal preconditioners that compete very well with the Jacobi iteration. Our proposal is to modify these iterations in order to solve the associated system in sequential form updating the known components of the solution.

In order to do this we need express the corresponding iteration as a splitting scheme and study the corresponding convergence properties. After that, a simulation will allows us to know how these preconditioners behave as well as compare them with classic preconditioner in this area.
Related Work and Justification

Computed solutions of systems of linear equations are one of the key tools in scientific computing with uncountable applications in mathematics and other sciences. There are two major approaches to solving system, direct methods and iterative methods.

In the area of iterative methods for linear systems, preconditioning became many years ago a very important tool for accelerating convergence toward a solution. It is worthy to mention that iterative methods and preconditioner work very well for sparse system, where direct methods generate a fill in, that can not be handled for large matrices. At this point researchers have been developing and using preconditioners that work well for specific problems.

Since the preconditioner introduced by Tarazaga and Cuellar [TC], can compete very well with the Jacobi iteration, the purpose of this work is to use a similar idea as the one used by the Gauss-Seidel iteration. In other words the goal of this work is to solve the basic iteration by finding the values of the components one by one and updating the solution vector with the new known information. This technique works very well for the Gauss-Seidel iteration and we are expecting similar results for the new methods based on the Tarazaga-Cuellar iterations.

We want to point out that not only is better convergence expected but also to enlarge the class of matrices for which we can prove convergence. In this way we are expecting a couple of new iteration and preconditioner that may compete well with Gauss-Seidel.
There is a profuse literature for iterative methods and preconditioners, some of the classic references are [S], [W] and [A].
Planned activities

Reading of research material

I will read basic chapters of a couple of books on iterative methods and papers related to the subject, some of them are mentioned in the references above, the complete list will be provided by my adviser.

Developing of tools

I will develop in Matlab the necessary codes to compute new preconditioning matrices and to perform simulations that allow me to compare the new preconditioner with known ones. Particular emphasis will be placed upon sparse matrices. Results of the simulations will be a section in the Thesis.

Regular meetings with my advisor

I will have regular meetings with my advisor, in principle weekly, to talk with him about the progress of my research and to have answers for the problems I may found during my work.

Writing a monograph

At the end of my research I will collect all the results in a monograph as well as the tool I developed to perform my research and I will include the conclusions of my research.
End Result

At the end of this research I expect to be able to understand how iterative methods and preconditioners work to compute solutions to linear system. Also I expect to be able to generate the new iterations and compare their performance to known methods. I also expect to prove basic properties of the new iterative methods.

A monograph containing all the results will be also an end result of this project.
References


