A Tale of Three Calculators: Problem Solving with Graphing Calculators on the UIL Mathematics Exam

A PROPOSAL for a PROJECT in MATHEMATICS

by

JAMES STEVEN DAUBNEY

COMMITTEE MEMBERS

Dr. George Tintera  
Committee Chair

Dr. Elaine Young  
Committee Member

Dr. Nadina Duran-Hutchings  
Committee Member

Dr. Blair Sterba-Boatwright  
Chair, Department of Mathematics and Statistics

Date  May 5, 2008
ABSTRACT

The Texas University Interscholastic League (UIL) Mathematics Examination is given every March to secondary students. It includes topics from algebra I and II, geometry, trigonometry, math analysis, analytic geometry, pre-calculus and elementary calculus. Contestants are allowed the use of any commercially available silent hand-held calculator. The most common high school calculator is the TI-83/84, and to date it is also the most popular on the exam. However, a growing number of participants have been using the TI-89 CAS calculators. The even newer TI-Nspire CAS calculators will be available for next year's examinations. Yet many students and educators are unfamiliar with these CAS calculators and their capabilities. Accordingly, this project will attempt to familiarize the education community with all three of these calculators (the TI-84 Plus, the TI-89 Titanium, and the TI-Nspire CAS), all of which are being actively marketed, by juxtaposing their utilization on the 2008 District II UIL Mathematics Examination. The author will produce a series of lessons which will demonstrate how to solve the problems presented on the examination using the Texas Instrument calculators which are currently marketed for education. Side-by-side comparisons of algorithms, entries, announcements, menus, graphs and tables will enhance a reader's capacity to use a variety of calculators in this context. It is the author's hope that this will result in both a greater understanding of the calculators' broad capabilities and improvement of students' mathematical problem-solving abilities.
INTRODUCTION

The University Interscholastic League (UIL) is the state-wide organization for public elementary and secondary interschool competition in Texas. The UIL Mathematics Examination was first introduced in 1991 and presently consists of 60 questions with a 40-minute time limit (Texas Competitive Mathematics, 2008). It is currently based on the secondary mathematics Texas Essential Knowledge and Skills (TEKS), including concepts from algebra I and II, geometry, trigonometry, math analysis, analytic geometry, pre-calculus and elementary calculus. The district examinations are given annually in the spring, however, many invitational meets occur throughout the year. There are two releases annually of UIL exams prepared for invitational meets, plus several more annually from the Texas Math and Science Coaches Association (TMSCA). Contestants are allowed the use of any unmodified commercially available silent hand-held calculators that do not require auxiliary electric power. They are allowed to maintain any equations or programs that they may have stored in their calculator’s memory, and they are allowed to bring a spare calculator. Small, hand-held computers are not permitted (UIL, 2007). The most popular calculator used is the TI-84 Plus, although increasingly more contestants are using the TI-89 Titanium. A new graphing calculator, the TI-Nspire CAS, will also be eligible for the examination.

By focusing on problem solving in the UIL Mathematics Examination and by juxtaposing strategies on these three calculators, the author hopes to produce educational materials which will help students achieve greater competency with
graphing calculators (including the newer CAS models) and problem-solving techniques.

The purpose of this project is to produce classroom lessons and activities for use with secondary educators and students involved in UIL Mathematics, as well as advanced secondary mathematics students who use graphing calculators in their studies. These lessons and activities will meaningfully engage educators and students in problem solving and graphing calculator usage, which may extend beyond the UIL Mathematics examination and into life-long problem-solving strategies. This project is based on the following guiding principles:

1. Mathematics problem-solving can be effectively taught and learned using graphing calculators.
2. The UIL Mathematics Examination offers a wide variety of problems suitable for teaching mathematics problem-solving at the secondary level.
3. Juxtaposing different calculator strategies for the same problem is an efficient method of teaching calculator logic and usage.
4. Computer algebra system (CAS) graphing calculators can increase mathematical understanding.
5. Multiple representations offered by graphing calculators enhance student understanding of mathematical principles and aid in problem-solving.
RELATED WORKS AND JUSTIFICATION

Calculators may be classified as four-function, scientific, graphing, and computer algebra system (CAS). The four-function calculator adds, subtracts, multiplies and divides. The scientific calculators employ scientific notation, floating point arithmetic, logarithmic functions, trigonometric functions, exponents and roots, and constants such as pi and e. Deluxe scientific calculators also include hexadecimal and binary calculations, complex numbers, fractions, and statistical and probability calculations. Graphing calculators have all of the above functions, plus they are capable of plotting graphs, solving systems of equations, and are often programmable. CAS calculators are graphing calculators which store and display values symbolically rather than floating-point numbers. They feature automated symbolic manipulation, as well as various algebraic and calculus based functions, including symbolic limits, derivatives, and integrals.

Virtually all calculators now employ liquid crystal displays (LCD), but some can be further differentiated by larger QWERTY keyboards, similar to typewriters. Another differentiating factor is input, which may be either reverse Polish notation (RPN) or algebraic. Algebraic input follows mathematical expressions from left to right. However, RPN inputs require that operands are entered immediately preceding their operators. Presently, only Hewlett-Packard mass markets calculators with predominantly RPN input.

Battery-powered pocket calculators were commercially introduced circa 1970 and the first scientific pocket calculator, the HP-35, appeared in 1972 at a cost of $395. During the 1970's, as calculator prices fell, calculator sales grew,
while sales of slide rules and math tables plummeted. The NCTM advocated broad student access to scientific calculators (Kennedy, 2002), and the College Board acquiesced, by allowing, but not requiring, scientific calculators for the first time on the 1983 AP calculus examinations. The experiment lasted only two years, as the results were poor and the construct validity of the examination was called into question (Kennedy, 2002).

It is often said that assessment drives the curriculum, and in 1989 the National Council of Teachers of Mathematics (NCTM, 1989, overview) reiterated this position in the *Curriculum and Evaluation Standards for School Mathematics* by pointing out that appropriate calculators should be available to all students at all times, and calculators should be fully integrated into the teaching and the testing of mathematics.

The mass marketing of graphing calculators in 1988 renewed this debate, and by 1993 scientific calculators were required on the AP examinations. Beginning in 1995, graphing calculators which could graph a function, solve an equation, compute a numerical derivative at a point, and compute a definite integral numerically were required. However, a portion of the multiple-choice test was kept calculator-free. When CAS calculators became widely available and were approved for testing in 1999, the College Board made a portion of the free-response section calculator-free as well (Kennedy, 2002).

Calculators have been allowed, but not required, for the Scholastic Aptitude Test (now the SAT) since 1994. This policy allows any type of
calculator, from basic four-function calculators to graphing calculators with symbolic algebra capabilities (Dion, Harvey, Jackson, Klag, Liu & Wright, 2002).

The UIL added the Mathematics Examination for Texas secondary students in 1991 and recently changed its rules to allow any commercially available silent hand-held calculator that does not require auxiliary electric power (Texas Competitive Mathematics, 2008).

The advent of the CAS graphing calculator has renewed the technology controversy in mathematics instruction and assessment (Zbiek & Schoaff, 2002). Although it is a commonly held belief that the use of CAS in secondary school mathematics causes atrophy of paper and pencil symbolic manipulations skills, a survey of numerous studies points to the contrary (Heid, Blume, Hollebrands, & Piez, 2002). In fact, students who have utilized CAS based instruction have surpassed their counterparts even in paper and pencil skills. CAS gives students greater flexibility in problem solving by offering them a greater range of strategies while developing deeper understanding of the concepts of function, variable, and parameter (Heid, et al., 2002). Mathematics is really about reasoning and problem solving. The calculations, numerical or algebraic, are only a means to an end.

CAS allows students to learn precalculus and calculus even though they may not have mastered algebra (Mahoney, 2002). CAS calculators solve equations symbolically, and have the potential to cause a major paradigm shift in teaching mathematics, just as scientific and graphing calculators have done before.
When using CAS and most graphing calculators, students can seamlessly transition between symbolic, numeric and graphical representations (Pierce & Stacey, 2002). "Student's engagement with, and ownership of, abstract mathematical ideas can be fostered through technology. Technology enriches the range and quality of investigations by providing a means of viewing mathematical ideas from multiple perspectives" (NCTM, 2000, 24).

Calculators can change the way we teach. Bert K. Waits reports ten fundamental activities done with hand-held visualization technology in the classroom work of students in the Calculator and Computer Precalculus project (involving more than 1,000 schools in the USA). These activities are:

1) Approach problems numerically.
2) Use analytic algebraic manipulations to solve equations and inequalities and then support using visual methods.
3) Use visual methods to solve equations and inequalities and then confirm using analytic algebraic methods.
4) Model, simulate and solve problem situations.
5) Use computer generated visual scenarios to illustrate mathematical concepts.
6) Use visual methods to solve equations and inequalities which can not be solved or are impractical using analytic algebraic methods.
7) Conduct mathematical experiments, and make and test conjectures.
8) Study and classify the behavior of different classes of functions.
9) Foreshadow concepts of calculus.
10) Investigate and explore various connections among different representations of a problem situation.

(Carvalho, 1996, citing Waits, 23).

The National Council of Teachers of Mathematics advocates introducing, teaching, and assessing mathematics through problem-solving (NCTM, 2000, 334). The Texas UIL Mathematics Examination provides a wealth of problems on
a 60-question examination designed to test knowledge and understanding in the areas of algebra I and II, geometry, trigonometry, math analysis, analytic geometry, pre-calculus and elementary calculus. These are aligned with Texas Essential Knowledge and Skills (TEKS) for secondary students (Texas Education Agency, 2007). A total of seven tests are generated per year and are released to the public (Texas Competitive Mathematics, 2008). The rules allow almost any hand-held calculator. Accordingly they constitute an excellent problem bank for teaching problem solving with technology.

The world calculator market has four major manufacturers: Casio, Texas Instruments, Hewlett-Packard and Sharp. However, the North American educational market is virtually monopolized by Texas Instruments, so this project will utilize the three TI models which are actively marketed to the secondary and college market:

1. The **TI-84 Plus** (released in 1999 as the TI-83 and re-released in 2004 as the TI-84 Plus), which is by far the most common graphing calculator found in American secondary and post-secondary education today;

2. The **TI-89 Titanium** (released in 1998 as the TI-89 and re-released in 2004 as the TI-89 Titanium), which is the most widely used CAS calculator in North America, and which is used in many secondary and university advanced mathematics classes; and

3. The **TI-Nspire CAS**, (preliminary release in 2007, mass release in 2008, also released in a non-CAS version), which is heralded as
the next generation calculator of the future. It is also equipped with
dynamic geometry and spreadsheet capabilities (Woerner, 2007).

Calculator techniques will be learned by solving problems. Problems
taken from the 2008 UIL Mathematics Examination will be solved simultaneously
utilizing all three calculators, juxtaposing multiple representations, so as to build
upon the schemas of most educators and students with respect to the TI-84,
while introducing new methods and algorithms characteristic of the newer CAS
calculators. A copy of the 2008 District II UIL Mathematics examination is
attached as Appendix A.

Vygotsky's zone of proximal development represents the difference
between what a person can do with and without help; that is, just beyond the
learner's current competence. Activities such as associations, imitations and
manipulations are a necessary part of concept formation and can bridge the gap
between mathematical activities and the formation of mathematical concepts,
thereby complementing and building upon existing abilities (Berger, 2005).
Students and educators who may already have a working knowledge of the TI-
83/84 can build upon that knowledge while learning analogous strategies on the
TI-89 and Nspire CAS calculators.

Skemp and others have posited that mathematical understanding can be
bifurcated into instrumental understanding and relational understanding. The
former involves the mere mechanical application of rules and formulas which
students may not be able to apply outside the narrow context in which they were
learned; the latter necessitates deeper understanding of mathematics principles,
and while harder to learn it is easier to remember and apply to other situations (Skemp, 1987). Many would argue that teaching problem solving with calculators would be a quintessential case of instrumental teaching, but the author takes the contrary position. Problem solving with graphing calculators, particularly CAS graphing calculators, necessitates a fundamental understanding of basic mathematic principles, and with practice this knowledge is organic; that is, it nurtures its own growth and development. Problem solving with calculators promotes relational understanding, with applications which transcend multiple representations of problems.

This project is appropriate for a project for a master's degree because there is a need for educators and students to acquire greater technology and calculator skills in accord with the NCTM guidelines. Specifically, many educators and students haven't realized the full potential of graphing calculators, and even fewer have adequate knowledge of the capabilities of CAS calculators, especially the new Nspire CAS, which was first released in 2007.

PLANNED ACTIONS

This project begins with a detailed study of the 2008 UIL District II Mathematics Examination. The author will develop problem-solving strategies for each of the three calculators for each of the thirty odd-numbered problems. The author will then document these strategies using step-by-step screen captures and accompanying explanations. These materials will be developed for educators and students who have access to at least one of these three graphing
calculators, and preferably more than one. Partial results will be presented to
secondary students at Tuloso-Midway High School in May of 2008 and at the
Coastal Council of Teachers of Mathematics annual mathematics conference,
(ME)² by the Sea, at Texas A & M University, Corpus Christi in June of 2008. It
will subsequently be used as training for UIL contestans and secondary
mathematics students in Algebra II and above.

TIMELINE

January – April 2008  Prepare project proposal
April 28, 2008        Distribute project proposal to committee members
May 6, 2008          Project proposal defense
March – April 2008    Research UIL examinations, calculator capabilities,
presentation formats and lessons
April – May 2008      Create TI-84, 89, and Nspire CAS presentations with
                      respect to the 2008 District UIL Examination
May 2008              Presentation at Tuloso-Midway High School
June 2008             Presentation at the (ME)² by the Sea Conference
July 2008             Distribute project to committee members
July 2008             Project defense
August 2008           Graduation

END RESULT INTENDED

This project will produce lessons and activities to assist educators and
secondary mathematics students to adopt new graphing calculator problem-
solving strategies to achieve better results on the UIL Mathematics Examination
and other examinations, such as the SAT and AP Calculus examination. The
lessons will incorporate student-centered, problem-based instruction, technology
and additional practice problems. They will provide detailed step-by-step calculator solutions for the thirty odd-numbered problems on the 2008 District II UIL Mathematics Examination. There will be a total of five lessons featuring six problems each. Alongside the calculator screen captures will be written instructions and explanations for each of the three featured calculators, the TI-84 Plus, the TI-89 Titanium and the TI-Inspire CAS. In addition to the written lessons, the same lessons will be produced in a multimedia format suitable for group presentations. Similar problems from other UIL examinations will be added for practice. These materials will be distributed via the Internet, mathematical conferences, technology in-service training, and UIL meetings. An example of some sample problems and their calculator solutions is attached hereto as Appendix B.

Results might include increased interest in calculator-assisted problem solving, and enhanced performance on UIL Mathematics examinations and other examinations permitting the use of CAS and non-CAS graphing calculators, such as the TAKS, SAT, ACT, and AP exams. It is hoped that this may also enhance students' success in college-level courses permitting CAS graphing calculators.
REFERENCES


1. Evaluate: \( 5! \times (3)^{-2} \times \sqrt{(3)^2} + 5 - 7 \)

(A) 52  (B) 38  (C) \( \frac{32}{7} \)  (D) \(-\frac{52}{7}\)  (E) 62

2. \( 889 + 555_8 - 2222_8 = \) \( \_\_\_\_\_\_\_11 \)

(A) 311  (B) 214  (C) 211  (D) 186  (E) 73

3. Find the value of \( k \) in the system of equations, \( 5x - 4y = 3 \), \( 3x + 2y = 1 \), and \( kx - 3y = 6 \).

(A) \(-4\frac{1}{11}\)  (B) \(-1\frac{7}{11}\)  (C) 6  (D) 12  (E) 15

4. The radius of a wagon wheel is 12 inches. The wheel makes 840 revolutions in a mile. If the wheel was changed to a wheel with a radius of 15 inches, how many revolutions would it make in a mile? (nearest whole revolution)

(A) 420  (B) 560  (C) 630  (D) 672  (E) 700

5. Renay Daykart plotted the points (3, 0), (5, 2), (0, 2) and (\(-2, 0\)). What is the area of the quadrilateral formed by these points?

(A) 8.5 units\(^2\)  (B) 10 units\(^2\)  (C) 11.5 units\(^2\)  (D) 12 units\(^2\)  (E) 14.5 units\(^2\)

6. A circle is inscribed in a triangle. The lengths of the sides of the triangle are 5 cm, 8 cm, and 11 cm. Find the diameter of the circle. (nearest tenth)

(A) 12.0 cm  (B) 7.9 cm  (C) 6.0 cm  (D) 3.1 cm  (E) 1.2 cm

7. Who was the mathematician that was extremely proud of his discovery for finding the volume of a sphere?

(A) John Venn  (B) John Napier  (C) Euclid  (D) Diophantus  (E) Archimedes

8. \( y^2 - 2xy + x^2 = 0 \) is an equation of a degenerate conic. Which of the following best represents this equation?

(A) point  (B) line  (C) parallel lines  (D) intersecting lines  (E) no graph

9. Ranger Smokey can see Ranger Bare's tower from his tower on a bearing of 285\(^\circ\). What is the bearing from Bare's tower to Smokey's tower?

(A) 195\(^\circ\)  (B) 105\(^\circ\)  (C) 75\(^\circ\)  (D) 35\(^\circ\)  (E) 15\(^\circ\)

10. Find the value of \( \tan \left( \arccos \frac{5}{13} + \arcsin \frac{12}{13} \right) \). (closest approximation)

(A) \( \frac{\sqrt{3}}{2} \)  (B) 1  (C) \(-1\)  (D) \(-\frac{\sqrt{3}}{3}\)  (E) \(-\sqrt{3}\)
11. Which of the following system of inequalities would be best represented by the shaded region shown?

(A) \( y \leq -x - 2 \) 
    \( y \geq 3x^2 + 2x + 1 \)

(B) \( y \geq 2 - x \) 
    \( y \geq 2x^2 - 3x + 1 \)

(C) \( y \geq -x - 2 \) 
    \( y \leq 3x^2 + 2x + 1 \)

(D) \( y \leq 2 - x \) 
    \( y \geq 2x^2 + 3x + 1 \)

(E) \( y \leq 2 - x \) 
    \( y \geq 2x^2 - 3x + 1 \)

12. Let \( f(x) = \frac{2x^2 + 5x + 11}{x^2 + 1} \). How many asymptotes does \( f(x) \) have?

(A) 0    (B) 1    (C) 2    (D) 3    (E) 4

13. Hi Lowe invested some money in a mutual fund. Over a four year period his investment earned 10% the first year, 12% the second year, 7% the third year, and 5% the fourth year. What is the average rate of return over the four years? (nearest hundredth)

(A) 8.50 %    (B) 8.47 %    (C) 8.44 %    (D) 8.41 %    (E) 8.38 %

14. If \( f(\theta) = \cos 2\theta \) then \( f'(\frac{2\pi}{3}) = \phantom{0} \).

(A) \(-\sqrt{3}\)    (B) \(-\frac{\sqrt{2}}{2}\)    (C) \(-\frac{\sqrt{3}}{2}\)    (D) \(-\frac{1}{2}\)    (E) \(-1\)

15. Find the angle of rotation, \( \theta \) (nearest degree), where \( 0^\circ < \theta < 90^\circ \), such that the conic \( x^2 - 5xy + 3y^2 + 2y + 10 = 0 \) contains no \( xy \) term in its equation.

(A) 26°    (B) 30°    (C) 34°    (D) 36°    (E) 39°

16. Sixty students, including Dick and Jane, are to be split into three classes of equal size. What is the probability that Dick and Jane will be in the same class, provided the split is done randomly? (nearest tenth)

(A) 33.0 %    (B) 32.2 %    (C) 27.7 %    (D) 21.5 %    (E) 10.9 %

17. Let \( (r + (r)^{-1})^2 = 4 \) and \( (r^3 + (r)^{-3}) = -2 \). Find \( r \).

(A) \(-3\)    (B) \(-1\)    (C) 0    (D) 1    (E) 3

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18. The geometric mean of 2828 and 8282 is ____ % of the arithmetic mean of 2828 and 8282. (nearest whole %)
   (A) 115 %  (B) 87 %  (C) 53 %  (D) 34 %  (E) 15 %

19. Set $A = \{2, 5, 10, 17, 26, 37, 50, \ldots\}$. Which of the following is not an element of set $A$?
   (A) 122  (B) 170  (C) 195  (D) 257  (E) 626

20. Forty students are in 8th grade at Elza Middle School. Sixteen of them play basketball, thirty-three of them play football, and fifteen of them play both football and basketball. How many of the students don't play either?
   (A) 4  (B) 5  (C) 6  (D) 8  (E) 11

21. If \( \frac{2x+1}{2x-1} + \frac{2x-1}{2x+1} \) is written as the mixed number \( \frac{A}{B} \), then \( B \) is?
   (A) 16  (B) 11  (C) 9  (D) 8  (E) 4

22. Which of the inequalities is best represented by the graph below?

   (A) $3x - 2y \geq -4$  (B) $2x + 3y \leq -1$  (C) $2x + 3y \geq 4$
   (D) $3x - 2y \leq -4$  (E) $x + 3y \geq 4$

23. Vector $v$ (3, 4) is parallel to vector $u$ (k, 5). Find k.
   (A) $\frac{3}{4}$  (B) $\frac{2}{5}$  (C) $\frac{4}{15}$  (D) $\frac{5}{12}$  (E) $-\frac{62}{3}$

24. In $\triangle PRS$, QT // RS, ST = x, PT = 9, QT = 4.5 and RS = x + 3. Find x.
   (A) 3  (B) 4.5  (C) 6  (D) 7.5  (E) not enough information given
25. Two ellipses circles, \((2x+1)^2 + (y-3)^2 = 16\) and \((2x-1)^2 + (y+3)^2 = 9\), intersect at two points. Find the slope of the line passing through the two points of intersection.

(A) \(-\frac{3}{4}\)  \hspace{1cm} (B) \(-\frac{2}{3}\)  \hspace{1cm} (C) \(\frac{1}{2}\)  \hspace{1cm} (D) \(\frac{2}{3}\)  \hspace{1cm} (E) \(\frac{3}{2}\)

26. Which of the following is not a one-to-one function:

(A) \(f(x) = (2x - 4)^{-1}\)  \hspace{1cm} (B) \(f(x) = -x^3 + 3x^2 - 2\)  \hspace{1cm} (C) \(f(x) = 2 \log 3x\)  

(D) \(f(x) = 3 - 5x\)  \hspace{1cm} (E) \(f = \{(-1, 2), (0, 4), (2, -4), (5, 6), (10, 0)\}\)

27. If \(y^2 = 7 - i\) and \(y^3 = 8 + 6i\) where \(y = a + bi\) then \(a + b\) equals:

(A) 20  \hspace{1cm} (B) 8  \hspace{1cm} (C) 2  \hspace{1cm} (D) 1  \hspace{1cm} (E) 0

28. \((1 + i)(2 + i)(3 - i) = ?\)

(A) 6 + i  \hspace{1cm} (B) 6 + 0i  \hspace{1cm} (C) 8 - 6i  \hspace{1cm} (D) 0 + 10i  \hspace{1cm} (E) 6 + 8i

29. A triangle is drawn as shown. Find the height, \(h\), if \(YZ = 20''\), \(m\angle XZY = 40^\circ\), and \(XZ = 15''\). (nearest tenth)

\[\text{A triangle with points Y, Z, and X.}\]

(A) 17.5''  \hspace{1cm} (B) 11.5''  \hspace{1cm} (C) 15.3''  \hspace{1cm} (D) 9.6''  \hspace{1cm} (E) 12.9''

30. The graph of \(r = 7 + 3\cos \theta\) is a(n) _______.

(A) limacon  \hspace{1cm} (B) circle  \hspace{1cm} (C) rose curve  \hspace{1cm} (D) lemniscate  \hspace{1cm} (E) spiral

31. If \(A = \begin{bmatrix} 3 & -6 \\ -9 & 12 \end{bmatrix}\) and \(A^{-1}\) is the inverse matrix of \(A\) then the determinant of \(A^{-1}\) is:

(A) \(-18\)  \hspace{1cm} (B) \(-12\)  \hspace{1cm} (C) \(\frac{1}{18}\)  \hspace{1cm} (D) \(\frac{1}{3}\)  \hspace{1cm} (E) 18

32. The operation "\(\oplus\)" is defined as \(x \oplus y = (x + y)^{(x-y)}\). Compute \((-1 \oplus -1) \oplus -1\).

(A) 0  \hspace{1cm} (B) \(-1\)  \hspace{1cm} (C) 1  \hspace{1cm} (D) \(-2\)  \hspace{1cm} (E) 2

33. If \(f''(x) = 60x^3 - 12x\) and \(f'(1) = 10\) and \(f(-1) = -6\), then \(f(1) = \) _______.

(A) 10  \hspace{1cm} (B) 4  \hspace{1cm} (C) \(-1\)  \hspace{1cm} (D) \(-2\)  \hspace{1cm} (E) \(-8\)

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34. Find the area (in square units) of the shaded region. (nearest tenth)

\[ \frac{\text{(A) } \frac{2}{3}}{\text{B) } \frac{3}{4}} \quad \text{C) 1} \quad \text{D) } 1 \frac{1}{4} \quad \text{E) } 1 \frac{1}{3} \]

35. Ali Befare pulls the following cards from a regular deck of cards and randomly places them face down, 2♥, 2♦, 3♥, 5♠, 5♥, 5♦, 9♠, & 10♦. Two cards are randomly selected and turned face up. What is the probability that the first card is a prime number and the second card is a ♦?

\[ \begin{array}{lllll}
\text{(A) } 9\% & \text{ (B) } 9 \frac{3}{8}\% & \text{ (C) } 10\% & \text{ (D) } 10 \frac{5}{7}\% & \text{ (E) } 15\%
\end{array} \]

36. Two girls and two boys are sitting randomly in a row. What are the odds that both boys are sitting next to each other and both girls are sitting next to each?

\[ \begin{array}{lllll}
\text{(A) } \frac{1}{4} & \text{ (B) } \frac{1}{3} & \text{ (C) } \frac{1}{2} & \text{ (D) } \frac{2}{3} & \text{ (E) } \frac{3}{4}
\end{array} \]

37. I am an abundant, but unhappy, cubic number. Which of the following can I be?

\[ \begin{array}{lllll}
\text{(A) } 729 & \text{ (B) } 512 & \text{ (C) } 216 & \text{ (D) } 125 & \text{ (E) } 64
\end{array} \]

38. If \( \frac{7}{8} \) of P is 50% more than Q and Q is .75 times R, then R is what part of P?

\[ \begin{array}{lllll}
\text{(A) } 1 \frac{3}{7} & \text{ (B) } 2 \frac{1}{8} & \text{ (C) } \frac{9}{16} & \text{ (D) } \frac{7}{9} & \text{ (E) } 1 \frac{3}{8}
\end{array} \]

39. Which of the following nets when folded will form a cube?

\[ \begin{array}{lllll}
\text{(A)} & \text{(B)} & \text{(C)} & \text{(D)} & \text{(E)}
\end{array} \]

40. Which of the following is a solution to \( |5 - 3x| - 1 \geq 2 \)?

\[ \begin{array}{lllll}
\text{(A) } 2 \frac{1}{3} & \text{ (B) } 2 \frac{1}{4} & \text{ (C) } 1 \frac{5}{9} & \text{ (D) } \frac{5}{9} & \text{ (E) } \frac{3}{5}
\end{array} \]

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41. A larger pipe can fill a pool one and one-half times as fast as a smaller pipe. It takes 6 hours to fill the pool when both pipes are opened. How long would it take the smaller pipe to fill the pool by itself?

(A) 7.5 hrs  (B) 10 hrs  (C) 15 hrs  (D) 20 hrs  (E) 22.5 hrs

42. \( \angle ABE \) is supplementary to \( \angle EBF \), \( \angle ABC \) is complementary to \( \angle CBD \), and \( \angle DBE \) is complementary to \( \angle EBF \). If \( m\angle EBF = 32^\circ \) and \( m\angle CBD = m\angle DBE = 40^\circ \), then \( m\angle ABC = ? \)

(A) 82\(^\circ\)  (B) 72\(^\circ\)  (C) 54\(^\circ\)  (D) 50\(^\circ\)  (E) 48\(^\circ\)

43. Adjacent dots on the grid are 1 cm apart when measured vertically and horizontally. Find the area of the figure shown.

(A) 17 cm\(^2\)  (B) 15 cm\(^2\)  (C) 11.5 cm\(^2\)  (D) 10.5 cm\(^2\)  (E) 10 cm\(^2\)

44. The roots of the equation \( x^3 + bx^2 + 1.875x + d = 0 \) are \(-0.5, 0.25, \) and \( R \). Find \( R \).

(A) \(-8\)  (B) \(-4\)  (C) \(-2\)  (D) \(-0.875\)  (E) \(-0.125\)

45. Find \( x \) from the given system of equations: \[
4x - 3y = 2 \\
3x + 4y = 5
\]

(A) \(\frac{14}{23}\)  (B) \(\frac{9}{14}\)  (C) \(\frac{9}{25}\)  (D) \(\frac{14}{25}\)  (E) \(\frac{23}{25}\)

46. The vertex angle of an obtuse isosceles triangle has a measure of \(120^\circ\) and the length of its base is 8 cm. Find the perimeter of the triangle. (nearest tenth)

(A) 18.5 cm  (B) 17.2 cm  (C) 13.9 cm  (D) 12.6 cm  (E) 12.0 cm

47. How many distinct solutions exist for \( 2\sin(\theta)\cos(-\theta) = 2\sin(-\theta)\sin(\theta) \), where \( 0 \leq \theta < 2\pi \)?

(A) 0  (B) 1  (C) 2  (D) 3  (E) 4

48. Find the remainder when \( f(x) = 4x^3 - 6x^2 + 5x - 1 \) is divided by \( x - 1 \).

(A) 6  (B) 4  (C) 3  (D) 2  (E) 0

49. Let \( f(x) = \frac{x^2 - 2x}{2x + 4} \) and \( s(x) \) be the slant asymptote of \( f \). Find the value of \( s(-4) \).

(A) \(-4\)  (B) \(-2\)  (C) \(-1\)  (D) 2  (E) 5
50. Two numbers, \( x \geq 0 \) and \( y \geq 0 \), exist such that the sum of the numbers is 36 and that the product of one number and the square of the other number is a maximum. Find the larger of the two numbers.

(A) 12  (B) 18  (C) 22  (D) 24  (E) 30

51. Evaluate: \( \int_{-a}^{a} 2x^3 - 3x + 1 \, dx \)

(A) 2a  (B) \( a^4 + 2a \)  (C) \(-6a\)  (D) \(3a^2 + a\)  (E) does not exist

52. Let \( E = \{2,4,6,8\} \) and \( O = \{1,3,5,7,9\} \). Two elements of set \( E \) are selected at random without replacement and one element is selected from set \( O \). What is the probability that the product of the three numbers selected is an even number?

(A) 0%  (B) 25%  (C) 50%  (D) 75%  (E) 100%

53. The I. C. Kold Parlor offers thirty different flavors of ice cream. How many different triple-scoop ice cream cones are possible at the Kold Parlor?

(A) 4060  (B) 4546  (C) 4960  (D) 5400  (E) 5456

54. Three and two-fifths billion is added to two and three-fifths million. The sum is divided by four and one-fifth thousand. The quotient is multiplied by fifty thousand five hundred five. The product is rounded to the nearest whole number. How many zeros are in the final result?

(A) 6  (B) 4  (C) 3  (D) 1  (E) 0

55. If \( x - y = 6 \) and \( xy = 8 \) then \( x^3 - y^3 = ? \)

(A) 8  (B) 72  (C) 91  (D) 296  (E) 360

56. The point \((-4, -5)\) is rotated 150 degrees counterclockwise about the origin. The coordinates of the point after the rotation is \( \text{____}. \) (closest approximation)

(A) \((-1.0, -2.3)\)  (B) \((1.3, 3.7)\)  (C) \((6.0, 2.3)\)  (D) \((2.3, -1.0)\)  (E) \((6.0, 8.3)\)

57. The sum of the real solutions of \( |5 + |4 - x|| = 9 \) is:

(A) 9  (B) 8  (C) 1  (D) 0  (E) -4

58. The function \( f(x) = 1 + 2\sin \left( \frac{x}{3} - 3 \right) \) reaches a maximum point at which of the following points? (closest approximation)

(A) \((4.93, 2)\)  (B) \((-5.37, 3)\)  (C) \((-2.47, 1)\)  (D) \((5.37, 1)\)  (E) \((-4.93, 3)\)
59. Robin Bancks collects nickels and dimes. She has 250 coins worth a total of $16.00. How many nickels does Robin have?

(A) 70   (B) 90   (C) 110   (D) 125   (E) 180

60. Determine the sum of the infinite series: \(0.225 + 0.00225 + 0.0000225 + \ldots\)

(A) \(\frac{7}{22}\)   (B) \(\frac{5}{22}\)   (C) \(\frac{3}{11}\)   (D) \(\frac{7}{11}\)   (E) \(\frac{22}{27}\)
1. B
2. D
3. D
4. D
5. B
6. D
7. E
8. B
9. B
10. C
11. E
12. C
13. B
14. A
15. C
16. B
17. B
18. B
19. C
20. C
21. E
22. D
23. A
24. A
25. D
26. B
27. C
28. E
29. E
30. A
31. C
32. A
33. D
34. E
35. D
36. C
37. C
38. D
39. A
40. E
41. C
42. B
43. D
44. A
45. E
46. B
47. E
48. D
49. A
50. D
51. A
52. E
53. C
54. B
55. E
56. C
57. B
58. B
59. E
60. B
Appendix B

Appendix B

Problem-Solving Strategies with Graphing Calculators
3. Find the value of $k$ in the system of equations, $5x - 4y = 3$, $3x + 2y = 1$, and $kx - 3y = 6$.

(A) $-4 \frac{1}{11}$  (B) $-1 \frac{7}{11}$  (C) 6  (D) 12  (E) 15

3A. SOLVE WITH THE TI-84 PLUS

Solve the first two equations as a system for $(x, y)$ using an augmented matrix in reduced row echelon form and then evaluate the third equation at $(x,y)$ and solve for $k$.

Edit matrix A.

Create a $2 \times 3$ matrix using the equation coefficients.

Put the matrix in reduced-row echelon form.

Identify matrix A as the object of the command.
The announcement.

Execute the rref command.

Store the answer matrix as matrix B

Locate the decimal to fraction command.

Change the matrix to fraction form.

The first row ($B_{2,3}$) gives the value for one $x$, the second row ($B_{1,3}$) gives the value for one $y$. 
Substitute the values of $x(B_{2,3})$ and $y(B_{1,3})$ from the answer matrix $B$ into the third equation and solve for $k$. 

\[
\begin{bmatrix} 0 & 1 & -1.818181... \\ \text{Ans}\text{Frac} \\
\begin{bmatrix} 1 & 0 & 5/11 \\ 0 & 1 & -2/11 \end{bmatrix} \\
\left(6+3\left[B\right](2,3)\right)/
\left[B\right](1,3) \\
12
\end{bmatrix}
\]
3B. SOLVE WITH THE TI-89 TITANIUM

Solve the first two equations as a system for \((x, y)\) using an augmented matrix in reduced row echelon form and then evaluate the third equation at \((x, y)\) and solve for \(k\).

Locate the Matrix submenu from the Math menu.

Locate the reduced-row echelon command.

Enter a 2 x 3 matrix by using brackets for the beginning and end of the matrix as well as for each row.

Locate the solve command in the Algebra menu.
Appendix B

Substitute the values of $x$ and $y$ into the third equation and solve for $k$.

```
**solve(k=5/11-3*2/11=6,k)**
```

$k = 12$
3C. SOLVE WITH THE TI-Nspire CAS

Solve all three equations as a system for (x,y,k) using the equation solver.

In the Calculator menu locate the Algebra submenu and the Solve command.

In the template menu locate the Systems of Equations template.

Specify 3 equations.
Execute the template command.

Enter the three equations and solve for $x$, $y$, and $k$. 

\begin{align*}
5x - 6y &= -3 \\
3x + 2y &= 1, x, y, k \\
kx - 3y &= -6
\end{align*}
8. $y^2 - 2xy + x^2 = 0$ is an equation of a degenerate conic. Which of the following best represents this equation?

(A) point  (B) line  (C) parallel lines  (D) intersecting lines  (E) no graph

8A. SOLVE WITH THE TI-84 PLUS

Factor the equation by hand. Then test that the factors are correct using the "=" test. Finally, graph the factors.

Enter the factored equation.

Find the equals sign in the test menu.

Enter the original equation. A value of "1" means true, a value of "0" means false.
Graph each factor.

Observe a single line.
8B. SOLVE WITH THE TI-89 TITANIUM

Factor the expression and find the zeros. Then solve for y and graph the function.

Locate the factor command in the Algebra menu.

Enter the expression and factor.

Enter the factored equation and solve for y.

Paste the result to the equation editor (Y=).
Graph \( y = x \) and observe that it is linear.
8C. SOLVE WITH THE TI-NSPIRE CAS

Solve for y using the equation solver. Then graph the function.

Locate the solve command in the Algebra menu.

Enter the equation and solve for y.

Paste the result to the equation editor (Y=).
Graph $y = x$ and observe that it is linear.
11. Which of the following system of inequalities would be best represented by the shaded region shown?

(A) \[ y \leq -x - 2 \quad \text{and} \quad y \geq 3x^2 + 2x + 1 \]

(B) \[ y \geq 2 - x \quad \text{and} \quad y \geq 2x^2 - 3x + 1 \]

(C) \[ y \geq -x - 2 \quad \text{and} \quad y \leq 3x^2 + 2x + 1 \]

(D) \[ y \leq 2 - x - 2 \quad \text{and} \quad y \geq 2x^2 + 3x + 1 \]

(E) \[ y \leq 2 - x \quad \text{and} \quad y \geq 2x^2 - 3x + 1 \]
11A. SOLVE WITH THE TI-84 PLUS

Select the inequalities based on vertices, slopes and intercepts and then verify by graphing.

In the Format menu select GridOn and LabelOn for clarity.

Enter the first inequality in the equation editor.

Verify that its graph corresponds to the diagram as to slope and intercepts.

Enter the second inequality in the equation editor.

Verify that its graph corresponds to the diagram as to vertex and intercepts.
Indicate less than and greater than in the equation editor by repeating Enter.

Verify that the correct region is double shaded in the graph.
11B. SOLVE WITH THE TI-89 TITANIUM

Select the inequalities based on vertices, slopes and intercepts and then verify by graphing.

From Home select the function editor.

Enter the inequalities for \( y_1 \) and \( y_2 \).

In the Style menu specify whether to shade the area above or below the curve.

In the Style menu specify whether to shade the area above or below the curve.
In the Window menu select appropriate bounds for the relevant portion of the curves.

Verify that the correct region is shaded in the graph.
Appendix B

11C. SOLVE WITH THE TI-NSPRIRE CAS

Select the inequalities based on vertices, slopes and intercepts and then verify by graphing.

Choose Graphs & Geometry

Type the function $y \leq 2 - x$.

Enter to graph the inequality.
Type the second inequality

Enter to graph it.

In the Window menu select Window Settings
Select an appropriate window.

Tab to OK to see closer view.

In the Points & Lines menu choose Intersection Points.

Enter and click on the two inequality graphs to see the intersection points.