Tenth Grade Mathematics:
Intervention Materials for Frequently Missed Objectives

by

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Style: APA
ABSTRACT

The Texas Assessment of Knowledge and Skills (TAKS) test is an integral part of every Texas student's life. In order to receive a high school diploma, students must master the concepts included in the Texas Essential Knowledge and Skills (TEKS) which are contained in the objectives of the TAKS test for all core subject areas. Mathematics and Science TAKS tests have the highest student failure rates and are a primary reason many students do not graduate from high school because this test is required for graduation. In the author's experience, many students miss passing the test by only a few questions. Across the state, in the local district, and at the high school targeted by this project, the three most frequently missed objectives for mathematics involve functions, geometry and measurement. The purpose of this project was to provide teachers with easy access to rich instructional materials to use during regular instruction and for TAKS preparation tutorials. The materials were created for these three objectives with consideration of learning styles and differentiated instruction. It is hoped that these intervention materials will allow students to master these objectives and perform successfully on correlated TAKS questions.
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INTRODUCTION

Of the many high-stakes tests that Texas students take throughout their thirteen years of education, only one test impacts whether or not a student graduates from high school. The Texas Assessment of Knowledge and Skills (TAKS) test must be passed at the eleventh-grade level in Mathematics, Science, Social Studies, and English in order for a student to earn a diploma. Since this test is an integral part of a student’s graduation, passing this test in all four subject areas is crucial. This project focused on tenth grade mathematics objectives in preparation for the eleventh-grade exit exam.

The high school mathematics TAKS tests contain ten objectives (see Table 1) covering the Texas Essential Knowledge and Skills (TEKS). The three most frequently missed tenth grade mathematics objectives across the state are objectives two (functions), six (geometry) and eight (measurement and similarity). This trend is also reflected in the test scores for the author’s district and high school. In the author’s experience, many students are extremely close to passing this exam and often miss meeting the target score on the TAKS test by only a few questions. Mastering the most difficult and frequently missed objectives of TAKS can greatly affect a student’s outcome on this test. Instructional materials reinforcing these objectives will give students a greater chance of passing the test.
<table>
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<td>Measurement</td>
<td>7</td>
</tr>
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<td>9</td>
<td>Percents, Proportions, Probability and Statistics</td>
<td>5</td>
</tr>
<tr>
<td>10</td>
<td>Mathematical Processes and Tools</td>
<td>9</td>
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</tbody>
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Table 1: High School Mathematics TAKS Objectives (TEA, 2007)

The concepts in objectives six and eight are not formally studied in high school, but are foundations for the learning of concepts in previous, current and future classes. For this reason, other mathematics classes that students take, from Algebra I to Calculus, are affected by the lack of understanding of the content contained in these objectives. The problem occurs because eighth grade students do not learn the TEKS within these objectives deeply enough during regular classroom instruction. Therefore, it is difficult for them to retain and transfer the content from the eighth grade TEKS to subsequent mathematics courses and higher-level objective-based assessments.

The purpose of this project was to produce classroom lessons and activities as part of an intervention packet for teachers to help students master the targeted mathematics TAKS objectives. These lessons and activities were designed to meaningfully engage students in learning the targeted objectives both in the classroom and in TEKS review tutorials. The author hopes that the
project curriculum will leave a lasting impression on students and help them perform successfully on the TAKS test.

This project was based on the following guiding principles:

1. Mathematically rich instructional materials should be used to address student learning styles and diverse abilities.

2. These materials should help students retain and transfer the content of the three frequently missed objectives in order to perform successfully on correlated TAKS questions.
LITERATURE REVIEW

Stutz (2007) reported that a record 40,182 (16%) students did not graduate from high school because they failed one or more portions (Mathematics, English, Science and Social Studies) of the exit-level TAKS test. Only 50% of tenth graders and 69% of eleventh graders passed all subject areas in the state (Stutz, 2007). This high-stakes test impacts a student's future at every public high school in Texas.

The three most frequently missed mathematics TAKS objectives across the state are two, six and eight. The most frequently missed objective was eight, which covers measurement and similarity (mainly from eighth grade TEKS). Only 54.9% of tenth grade students answered questions for this objective correctly. Objective two addresses properties and attributes of functions (Algebra I TEKS) and had a 57.3% success rate. Objective six includes geometric relationships and spatial reasoning (mostly eighth grade TEKS) and had a 58.5% success rate (TEA, 2007).

The target district had only 50.9% of tenth grade students answer objective eight questions correctly and the target high school had only 52.1% successful. Objective two had a success rate of 56.4% and 54.6% respectively. Objective six was passed at the district and high school with 56.8% and 59% success rates respectively (see Table 2). Since students state-wide and locally follow the same trends, there was definitely a need for intervention in these areas.
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<tbody>
<tr>
<td>8</td>
<td>54.9%</td>
<td>50.9%</td>
<td>52.1%</td>
</tr>
<tr>
<td>2</td>
<td>57.3%</td>
<td>56.4%</td>
<td>54.6%</td>
</tr>
<tr>
<td>6</td>
<td>58.5%</td>
<td>56.8%</td>
<td>59%</td>
</tr>
</tbody>
</table>

Table 2: Percent of Tenth Grade Students Who Answered TAKS Questions Correctly by Objective. (TEA, 2008)

Students take many benchmark assessments throughout the year and this data as well as data from previous TAKS tests is analyzed so that each teacher and student knows of individual weaknesses. The trends are usually the same between the benchmark and the actual TAKS data. Researchers and educators have looked for various causes for this gap in students' knowledge from eighth grade to high school. One of the most common reasons for this gap is students who take Algebra I in eighth grade rather than the typical eighth grade mathematics class. A local teacher educator focus group reported that students who take Algebra I in eighth grade and then Geometry in ninth grade really struggle on the tenth and eleventh grade TAKS exams. They usually miss the eighth grade objectives, especially geometry, probability and statistics questions (Del Mar College, 2003).

This gap has caused students to have misconceptions about many of the eighth grade TEKS during later mathematics classes since these TEKS were not covered during Algebra I in the eighth grade. Many teachers use "re-teaching" as a tool for battling these challenging objectives. According to the Georgia Department of Education, re-teaching should focus on the skills and knowledge
that have not yet been mastered, using research-based instructional strategies (Cox, 2007). From the author's experience, many teachers re-teach with the same methods that they originally used to deliver the instruction. Since the material is presented the same way, students who did not understand the material the first time usually do not understand the material the second time. When it comes to addressing students who need intervention, differentiated strategies may improve learning. Using differentiated strategies means that teachers do not teach with the same method every time. Teachers can use different methods in order to help students with different learning styles. Many students who need intervention struggle to learn concepts because they are not able to grasp abstract concepts (Glencoe, 2005). Interventions should help students better understand the material, which can lead to success on assessments covering these objectives.

Many schools throughout the state have tried to develop strategies to increase student success on this exam. The administration at the targeted high school has decided to use a pull-out program during the 2007-2008 school year. Other schools throughout Texas also use this method of taking students out of elective classes and having them attend a TAKS tutorial class (Nelsen, 2007). Typically in this type of program, students are taken out of an elective class and put in a classroom with a teacher that is not certified in mathematics and the students are usually just given worksheets. The author believes that if the teachers teaching these TAKS pull-out classes had easy access to more mathematically robust and cognitively engaging instructional materials, the
students would be more likely to gain a deeper understanding of these critical objectives during the tutorials.

Soon the TAKS test will be phased out and replaced with End-of-Course exams for each core class. This would require students to pass four exams every year for three years in order to graduate. Many educators fear this will be an even larger obstacle for students to overcome (Cromer, 2007). Even though the TAKS exams are being phased out, reinforcing these three objectives would still benefit high school students on any type of exam that also covers material contained in these objectives.

Many textbook publishers and outside tutoring services have discovered the importance of the TAKS test and have developed lessons to teach the TEKS contained in the TAKS objectives. One example of this is Study Island (2008). This website allows people to buy a program that students can use at their own pace to help them master the TEKS that are tested on the TAKS. The problem with many of these materials is that many developers have not been in the classroom recently or, in some cases, at all. Without having recent classroom experience, it is difficult for someone to develop lessons that will actually work well in today’s classroom. For changes in mathematics education to be effective, it is crucial that a mathematics curriculum is developed that will impact the current philosophies in education and provide instruction with methods that are appropriate to the population at the present time.

Differentiated instruction is one way to increase student achievement. Dobush (2007) explains, “Differentiated instruction is teaching with student
variance in mind. It means starting where the kids are rather than adopting a standardized approach to teaching that seems to presume that all learners of a given age or grade are essentially alike. Thus differentiated instruction is “responsive” teaching rather than “one-size-fits-all” teaching” (1). This strategy could also be applied to lessons and activities that are geared toward student success on standardized achievement tests such as TAKS. By realizing that students do not learn from the same methods, teachers can differentiate lessons and reach more students. As a result, there would be a larger student success rate at standardized exams.

Having mathematically rich lessons and activities is another way to boost student success. According to the National Council of Teachers of Mathematics (NCTM, 2006), mathematically rich lessons involve challenging, coherent, well-presented and engaging mathematics. In order for materials to be considered mathematically rich, they need to be designed to contain these qualities. An aspect of lesson design that is critical to address is learning styles. Every student does not learn with the same method and using diverse methods is necessary to reach more students. NCTM (2007) has also researched learning styles and found that “teachers know that multiple representations of mathematical ideas have the power to transcend language barriers and learning styles, but textbooks frequently just show one representation” (1). Teachers can step in and supplement the textbook content with multiple representations but often do not which leaves many students lacking in understanding. Another aspect of lesson design to be taken into account is diversity in student ability levels. Students of
many different ability levels are often in the same classes and the teacher is left to adjust instruction so that the lowest level student can grasp the material but the highest level student is still challenged. This is often a difficult task for teachers because there can be a large gap between the lowest and the highest student abilities. Creating student-centered lessons is one key to solving this problem. Cognitively-high students can discover information and solutions on their own and lower level students can be guided and still create some learning on their own. By differentiating instruction and allowing students to discover some concepts on their own, more students can be successful.

Creating instructional materials that aid in retention and transfer of knowledge is another integral part of lesson design and can have a residual effect on student learning. Students must be able to retain the information that they learn over time. In order for students to retain information, the material needs to be presented and learned in a way that is meaningful and relevant to the students so they will remember. The tenth grade TAKS exam covers material from 8th to 10th grade, so it is crucial that students retain information from previous years’ classes. The final aspect is transfer. Students must be able to take what they have learned about these concepts and transfer it to new situations. Many times students will see new problems that cover material that they have already learned, but it is presented in a different manner and they do not know how to solve them. With a deep understanding of the concepts, students should be able to retain and transfer the material to new situations with ease.
METHODOLOGY

This project began with researching the TEKS contained in the three most frequently missed TAKS objectives (2, 6, and 8) and the methods in which these TEKS were taught. Problems, examples, and activities were carefully chosen to have the greatest impact on student understanding of these critical TEKS. The games were designed to serve as a review of these TEKS in a fun, yet meaningful manner.

Lessons from this project were implemented in a tenth grade classroom and adjusted to be easier for students to understand. These lessons were also presented at the (ME)^2 by the Sea annual conference to current and future teachers. The teachers attending the sessions were asked to give qualitative feedback on the notes and activities presented. Feedback was used to adjust the curriculum to be more successful for student learning and user-friendly. The attendees were asked the following questions:

1. Would you be able to implement this curriculum in your classroom? If not, why not?
2. In your opinion, how effective is this curriculum in targeting these three weak objectives?
3. Which piece of this curriculum do you think you would use in your classroom (content notes, activities, games) and why?
4. Do you believe that these games could help students better retain and transfer this content to new situations?
5. What ideas do you have (if any) to improve this curriculum?
RESULTS

Student misconceptions and gaps in these three critical objectives are important to fill. The content notes, activities and games are all formatted to help students with the three targeted objectives. This curriculum contains content notes and review games for each of the three objectives and can be broken down by TEKS or used as a packet by TAKS objective. The curriculum is easy to mold and change for students of various ability levels. Students with higher levels of skill can use this curriculum with little or no help, while students with lower levels of skill can still use these materials with a little assistance from the teacher or a peer. Also included with this curriculum is a pre-test and post-test to assess student understanding before using the curriculum as well as after the curriculum has been implemented.

Lessons from this curriculum were distributed to teachers in the author’s district as well as to teachers attending the (ME)$^2$ by the Sea conference. It is the author’s hope that high school teachers will use this curriculum to address misconceptions that students have about the content in these three objectives.

This curriculum will be presented to the author’s district in August 2008 for implementation during the 2008-2009 school year. It is the author’s hope that tenth grade teachers throughout the region will use these materials to strengthen students in these critical areas.

In the future, the author plans to continue to produce more activities and games for these objectives and add them to this curriculum. The author hopes to be able to share these materials at other conferences also.
SUMMARY

In summary, the research for this project focused on the three most missed TAKS objectives for tenth grade students. This project is an intervention curriculum that can be used as one large packet or individually as many small lessons. It is the intention of the author to share this curriculum with other teachers in the district with the hope that it will strengthen students’ understanding of these critical TEKS and have a residual effect on students’ learning of future concepts.
ACKNOWLEDGEMENTS

I would like to thank Dr. Nadina Duran-Hutchings first and foremost for all of your help and support throughout the course of this project. Your guidance has really helped me complete this project and discover new ways of thinking and teaching. I really appreciate all of your help and support. I would also like to thank all of the other professors that have helped me work on this project and my education in general. Dr. Elaine Young, thanks for convincing me to go this route with my master's degree. I am really glad that I did!

Also, I would like to thank my family. Mom and Dad, you have always been there for me and supported me in all of the decisions that I made in my life. I am truly thankful for having such wonderful parents in my life. Angela, Mike, Teya and Jacob, thank you for your constant support and love. Angela you have showed me that anything is possible and I love you. To all of my friends that have encouraged me and helped me through these last two years, I really appreciate and love each of you. Traci, thanks for all the lunch and dinner dates! You have really kept me sane through this process. Katie, you have been an amazing friend and I don’t know what I would do without you!
REFERENCES


Cromer, B.K. (2007, April 18). Teachers are guarded on changes to testing. Fort Worth Star-Telegram (TX), Retrieved February 19, 2008, from Newspaper Source database.


APPENDIX A

LETTER TO TEACHERS
Dear Fellow Mathematics Teacher:

TAKS objectives 2, 6 and 8 are the most missed tenth grade objectives across the state. The following materials target these three critical objectives. These materials are not meant to be “one-size-fits-all”, instead they are meant to be a spring-board to address misconceptions about these objectives. I hope that you find these lessons useful for your classroom. All of these lessons and games were made using Microsoft Word and Excel and are completely editable. Please feel free to change and adapt these as you see fit for your classroom.

These lessons can be used as TAKS remediation, or whenever you find them useful in your curriculum. These lessons contain eighth grade and Algebra I TEKS as seen in objectives 2, 6, and 8. These lessons are grouped by TAKS objective, but can be used in any order that you see fit.

I hope that you will see the same successes that I have seen with these materials in your students and classrooms. Please let me know any suggestions, improvements, and successes that you have from using these materials in your classroom.

Sincerely,

Jennifer Jackson

ejackson@ccisd.us
APPENDIX B

OBJECTIVE 2 MATERIALS
Objective 2

Content Notes
Linear and Quadratic Functions

TEKS:
A.2(A): The student uses the properties and attributes of functions. The student is expected to identify and sketch the general forms of linear (y=x) and quadratic (y=x²) parent functions.

A.4(C): The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. The student is expected to connect equation notation with function notation, such as y = x + 1 and f(x) = x + 1.

Vocabulary:
equation notation
function
function notation
linear function
parabola
parent function
quadratic function
slope
y-intercept

A function is a relationship that assigns exactly one output value to each input value. A linear function is a function whose points lie on a line. The greatest power (exponent) on any variable in a linear function is 1. A quadratic function is a function whose greatest power is 2. The parent function (most general form) of a linear function is y = x. The parent function of a quadratic function is y = x². There are two different forms of notation used for functions. Equation notation is when functions are written with y = the function. Function notation is when functions are written with f(x) = the function. Both equation notation and function notation mean exactly the same thing. In other words, y and f(x) can be exchanged without changing any function values.

Linear functions can be written in the form y = mx + b, or f(x) = mx + b. This form of a linear equation tells you the slope (m) and the y-intercept (b).

Quadratic functions are usually written in the form y = ax² + c, or f(x) = ax² + c. Graphs of quadratic functions are U-shaped curves called parabolas. The coefficient “a” determines the wideness or narrowness of the graph as well as whether the graph opens up or down. The closer “a” is to zero, the wider the graph is. The larger the value
Objective 2

of "a", the narrower the graph is. If "a" is positive, the graph will open up and if the "a" is negative, the graph will open down. The "c" value will determine the y-intercept of the graph. A positive "c" value will move the parabola above the origin, while a negative "c" value will move the parabola below the origin.

**Example 1:**
Make a table of values and graph for the relation $y = 2x^2 + 3$.

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
</tr>
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<td></td>
</tr>
</tbody>
</table>

1. Is this relation a function? Why or why not? __________________________

2. Is this linear or quadratic? Why? __________________________

3. Is the graph wider or narrower than $y = x^2$? Why? __________________________

4. Is the graph above or below the origin? Why? __________________________

5. Find $f(6)$ __________________________ and $f(10)$. __________________________
Objective 2

Example 2:
Make a table of values and graph for the relation \( y = 2x - 2 \).

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

1. Is this relation a function? Why or why not?

2. Is this linear or quadratic? Why?

3. What is the slope? ________ y-intercept? ________

Example 3:
Given the following table, find the function that generates these values and graph the equation.

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-9</td>
</tr>
<tr>
<td>0</td>
<td>-6</td>
</tr>
<tr>
<td>1</td>
<td>-3</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

1. What is the slope? ________ y-intercept? ________

2. What is the function that represents this pattern? ________________
Generating Equations

**TEKS:**
A.3(A): The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations. The student is expected to use symbols to represent unknown variables.

A.3(B): The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations. The student is expected to look for patterns and generalizations algebraically.

**Vocabulary:**
- algebraic expression
- equation
- pattern
- variable

Letters, numbers, and symbols are used to create simplified models of mathematical problems. These models are called algebraic expressions. An equation is a mathematical sentence that states two expressions are equal. A variable is a symbol that stands for a number. A variable can either stand for the number of something or the amount of something. For example, $x$ can represent the number of hours you work and $y$ can represent the amount of money you make. To write an algebraic expression, choose variables to represent unknown values.

**Example 1:**
The length of a square is $(x - 3)$. Write an algebraic expression for the area.

**Example 2:**
Teya got a job selling shoes. She makes $6.25 an hour plus a 5% commission on the shoes that she sells.

1. What does $x$ represent in this situation?
2. What would $y$ represent in this situation?
3. Write an equation that models this situation.
4. How much would Teya make if she worked 10 hours and sold $400 worth of shoes?
Objective 2

*Example 3:*
Write an equation that models the data in the table. 

<table>
<thead>
<tr>
<th>X</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

*Example 4:*
Given the sequence, 1, 1, 2, 3, 5, 8..., answer the following questions.

1. What number comes next? 

2. What is the 10th term? 
Objective 2

Interpret Graphs and Situations

**TEKS:**
A.2(B): The student uses the properties and attributes of functions. The student is expected to identify mathematical domains and ranges and determine reasonable domain and range values for given situations, both continuous and discrete.

A.2(C): The student uses the properties and attributes of functions. The student is expected to interpret situations in terms of given graphs or create situations that fit given graphs.

A.2(D): The student uses the properties and attributes of functions. The student is expected to collect and organize data, make and interpret scatterplots (including recognizing positive, negative and no correlation for data approximating linear situations), and model, predict, and make decisions and critical judgments in problem situations.

**Vocabulary:**
- continuous
- discrete
- domain
- range
- scatterplot

There are many different ways to graph data. Depending on the type of data, you could have a continuous graph or a discrete graph. A continuous graph has connected lines or curves. All of the points that lie on the lines or curves are possible solutions to the situation. A discrete graph contains only distinct points as solutions. For example, if you are looking at a graph of the number of people that visit the library every day for a given week, your graph would be discrete. Since it is physically impossible to have ½ of a person visit the library, a continuous graph would not make sense.

Domain and range are two key aspects of any relation or function. The domain is the set of all input values for a relation or function. On the coordinate plane, the domain is the set of x-values (first coordinates). The range is the set of all output values for a relation or function. On the coordinate plane, the range is the set of all y-values (second coordinates).
Objective 2

**Example 1:**
Determine the domain and range of the given data set.
\{(-1, 4), (2, 0), (-3, 4), (0, 5)\}

Domain: ______________________________

Range: ______________________________

**Example 2:**
Write a possible scenario for the given graph. Make sure that you pay attention to the units on the axes.

```
```

**Example 3:**
Draw a graph of speed versus time that could fit the given scenario.

Jacob leaves his house riding his bike for 5 minutes to his friend Ryley's house. He then stops for 10 minutes to talk to him. He then leaves to go back home and arrives home 5 minutes later.

```
```
Example 4:
Use the following scatterplot of age (years) versus height (inches) to answer the questions.

1. What is the domain for this data?

2. What is the range for this data?

3. What type of correlation exists (if any) with this data?

4. Draw a line that fits as many of the points as possible.

5. Estimate the height of a 17 year old.
Solving Equations

**TEKS:**

**A.4(A):** The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. The student is expected to find specific function values, simplify polynomial expressions, transform and solve equations, and factor as necessary in problem situations.

**A.4(B):** The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. The student is expected to use the commutative, associative, and distributive properties to simplify algebraic expressions.

**Vocabulary:**

- associative property
- commutative property
- distributive property
- factor
- like terms

An algebraic expression is a combination of numbers, operations, and one or more variables. Mathematical properties and the order of operations are used to simplify expressions.

<table>
<thead>
<tr>
<th>Property</th>
<th>Description</th>
<th>Algebra</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commutative Property</td>
<td>You can add or multiply numbers in any order.</td>
<td>$a + b = b + a$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$ab = ba$</td>
</tr>
<tr>
<td>Associative Property</td>
<td>When you are only adding or multiplying, you can group any of the numbers together.</td>
<td>$a + b + c = (a + b) + c = a + (b + c) = abc = a(bc) = (ab)c$</td>
</tr>
<tr>
<td>Distributive Property</td>
<td>You can multiply a number by a sum or multiply the number by each number in the sum and then add.</td>
<td>$a(b + c) = ab + ac$</td>
</tr>
</tbody>
</table>
Objective 2

**Example 1:**
1. The width of a rectangle is 3 feet longer than twice the length. If the length is \( x \) feet, write an expression for the width of the rectangle.

2. Write an expression for the area of the rectangle. \( A = \) __________________

3. If the area is 65 ft\(^2\), what is the length of the rectangle? __________________

4. What is the width? __________________

**Example 2:**
A triangle has a height of \( x \) inches and an area of \( x^2 - 3x \) inches squared.

Find an expression for the base in terms of \( x \). __________________

If the area is 10 in\(^2\), what is the value of \( x \)? __________________

**Example 3:**
Simplify \( 4(2x + 3) - (x - 2)^2 \). __________________

**Example 4:**
The area of a rectangle is \( 3x^2 + 22x - 16 \). The length is \( 3x - 2 \). Find the width.

______________________________

**Example 5:**
Given the function \( f(x) = 2x^2 + 3x - 5 \), find \( f(4) \).

______________________________

**Example 6:**
Solve \( x^2 - x - 6 = 0 \).

______________________________
Objective 2

Content Notes Key
Linear and Quadratic Functions

**TEKS:**
A.2(A): The student uses the properties and attributes of functions. The student is expected to identify and sketch the general forms of linear \((y=x)\) and quadratic \((y=x^2)\) parent functions.

A.4(C): The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. The student is expected to connect equation notation with function notation, such as \(y = x + 1\) and \(f(x) = x + 1\).

**Vocabulary:**
equation notation
function
function notation
linear function
parabola
parent function
quadratic function
slope
y-intercept

A function is a relationship that assigns exactly one output value to each input value. A linear function is a function whose points lie on a line. The greatest power (exponent) on any variable in a linear function is 1. A quadratic function is a function whose greatest power is 2. The parent function (most general form) of a linear function is \(y = x\). The parent function of a quadratic function is \(y = x^2\). There are two different forms of notation used for functions. Equation notation is when functions are written with \(y = \) the function. Function notation is when functions are written with \(f(x) = \) the function. Both equation notation and function notation mean exactly the same thing. In other words, \(y\) and \(f(x)\) can be exchanged without changing any function values.

Linear functions can be written in the form \(y = mx + b\), or \(f(x) = mx + b\). This form of a linear equation tells you the slope (m) and the y-intercept (b).

Quadratic functions are usually written in the form \(y = ax^2 + c\), or \(f(x) = ax^2 + c\). Graphs of quadratic functions are U-shaped curves called parabolas. The coefficient "a" determines the wideness or narrowness of the graph as well as whether the graph opens up or down. The closer "a" is to zero, the wider the graph is. The larger the value
Objective 2

of “a”, the narrower the graph is. If “a” is positive, the graph opens upward and if “a” is negative, the graph opens downward. The “c” value will determine the y-intercept of the graph. A positive “c” value will move the parabola above the origin, while a negative “c” value will move the parabola below the origin.

**Example 1:**
Make a table of values and graph for the relation \( y = 2x^2 + 3 \).

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>11</td>
</tr>
<tr>
<td>-1</td>
<td>5</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>21</td>
</tr>
</tbody>
</table>

1. Is this relation a function? Why or why not? **Yes, this relation is a function because every input has a distinct output.**

2. Is this linear or quadratic? Why? **Quadratic because the highest exponent is 2. The graph is a parabola.**

3. Is the graph wider or narrower than \( y = x^2 \)? Why? **Narrower because the “a” value is 2.**

4. Is the graph above or below the origin? Why? **Above because the “c” value is positive 3.**

5. Find \( f(6) \) \( 75 \) and \( f(10) \). **203**
Objective 2

Example 2:
Make a table of values and graph for the relation \( y = 2x - 2 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-6</td>
</tr>
<tr>
<td>-1</td>
<td>-4</td>
</tr>
<tr>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

1. Is this relation a function? Why or why not? 
**Yes, this relation is a function because every input has a distinct output.**

2. Is this linear or quadratic? Why? 
**Linear, the highest exponent is 1. The graph is a straight line.**

3. What is the slope? 2  y-intercept? -2

Example 3:
Given the following table, find the function that generates these values and graph the equation.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-9</td>
</tr>
<tr>
<td>0</td>
<td>-6</td>
</tr>
<tr>
<td>1</td>
<td>-3</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

1. What is the slope? 3  y-intercept? -6

2. What is the function that represents this pattern? \( y = 3x - 6 \)
Generating Equations

**TEKS:**

**A.3(A):** The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations. The student is expected to use symbols to represent unknown variables.

**A.3(B):** The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations. The student is expected to look for patterns and generalizations algebraically.

**Vocabulary:**

- algebraic expression
- equation
- pattern
- variable

Letters, numbers, and symbols are used to create simplified models of mathematical problems. These models are called algebraic expressions. An equation is a mathematical sentence that states two expressions are equal. A variable is a symbol that stands for a number. A variable can either stand for the number of something or the amount of something. For example, $x$ can represent the number of hours you work and $y$ can represent the amount of money you make. To write an algebraic expression, chose variables to represent unknown values.

**Example 1:**
The length of a square is $(x - 3)$. Write an algebraic expression for the area.

$$(x - 3)^2 = x^2 - 6x + 9$$

**Example 2:**
Teya got a job selling shoes. She makes $6.25 an hour plus a 5% commission on the shoes that she sells.

1. What does $x$ represent in this situation? **The number of hours Teya works**

2. What would $y$ represent in this situation? **The amount of money Teya makes**

3. Write an equation that models this situation. $y = 6.25x + 0.05s$

4. How much would Teya make if she worked 10 hours and sold $400 worth of shoes? **$82.50**
Objective 2

**Example 3:**
Write an equation that models the data in the table. $y = 2x + 2$

<table>
<thead>
<tr>
<th>X</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

**Example 4:**
Given the sequence, 1, 1, 2, 3, 5, 8..., answer the following questions.

1. What number comes next? **13**
2. What is the 10th term? **55**
Objective 2

Interpret Graphs and Situations

**TEKS:**
A.2(B): The student uses the properties and attributes of functions. The student is expected to identify mathematical domains and ranges and determine reasonable domain and range values for given situations, both continuous and discrete.

A.2(C): The student uses the properties and attributes of functions. The student is expected to interpret situations in terms of given graphs or create situations that fit given graphs.

A.2(D): The student uses the properties and attributes of functions. The student is expected to collect and organize data, make and interpret scatterplots (including recognizing positive, negative and no correlation for data approximating linear situations), and model, predict, and make decisions and critical judgments in problem situations.

**Vocabulary:**
- continuous
- discrete
- domain
- range
- scatterplot

There are many different ways to graph data. Depending on the type of data, you could have a continuous graph or a discrete graph. A continuous graph has connected lines or curves. All of the points that lie on the lines or curves are possible solutions to the situation. A discrete graph contains only distinct points as solutions. For example, if you are looking at a graph of the number of people that visit the library every day for a given week, your graph would be discrete. Since it is physically impossible to have ½ of a person visit the library, a continuous graph would not make sense.

Domain and range are two key aspects of any relation or function. The domain is the set of all input values for a relation or function. On the coordinate plane, the domain is the set of x-values (first coordinates). The range is the set of all output values for a relation or function. On the coordinate plane, the range is the set of all y-values (second coordinates).
Objective 2

**Example 1:**
Determine the domain and range of the given data set.
\[\{-1, 4\}, \{2, 0\}, \{-3, 4\}, \{0, 5\}\]

Domain: \{-3, -1, 0, 2\}

Range: \{0, 4, 5\}

**Example 2:**
Write a possible scenario for the given graph. Make sure that you pay attention to the units on the axes.

*Answers will vary.* Possible answer: the height of a candle as it burns over 9 hours.

**Example 3:**
Draw a graph of distance from Jacob’s house versus time that could fit the given scenario.

Jacob leaves his house riding his bike for 5 minutes to his friend Ryley’s house. He then stops for 10 minutes to talk to him. He then leaves to go back home and arrives home 5 minutes later.

*Answers will vary.* See graph.
Objective 2

*Example 4:* Use the following scatterplot of age (years) versus height (inches) to answer the questions.

1. What is the domain for this data? \(\{5-15\}\)

2. What is the range for this data? \(\{36-65\}\)

3. What type of correlation exists (if any) with this data? *positive*

4. Draw a line through the origin that fits as many of the points as possible.

5. Estimate the height of a 17 year old. *Answers will vary. Between 65 and 70 inches.*
Objective 2

Solving Equations

**TEKS:**
A.4(A): The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. The student is expected to find specific function values, simplify polynomial expressions, transform and solve equations, and factor as necessary in problem situations.

A.4(B): The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. The student is expected to use the commutative, associative, and distributive properties to simplify algebraic expressions.

**Vocabulary:**
- associative property
- commutative property
- distributive property
- factor
- like terms

An algebraic expression is a combination of numbers, operations, and one or more variables. Mathematical properties and the order of operations are used to simplify expressions.

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</tbody>
</table>
Objective 2

**Example 1:**
1. The width of a rectangle is 3 feet longer than twice the length. If the length is $x$ feet, write an expression for the width of the rectangle.
   
   $2x + 3$

2. Write an expression for the area of the rectangle. $A = 2x^2 + 3x$

3. If the area is 65 ft$^2$, what is the length of the rectangle? 5 feet

4. What is the width? 13 feet

**Example 2:**
A triangle has a height of $x$ inches and an area of $x^2 - 3x$ inches squared.

Find an expression for the base in terms of $x$. $x - 3$

If the area is 10 in$^2$, what is the value of $x$? 5

**Example 3:**
Simplify $4(2x + 3) - (x - 2)^2$. $-x^2 + 12x + 8$

**Example 4:**
The area of a rectangle is $3x^2 + 22x - 16$. The length is $(3x - 2)$. Find the width.

$(x + 8)$

**Example 5:**
Given the function $f(x) = 2x^2 + 3x - 5$, find $f(4)$. 39

**Example 6:**
Solve $x^2 - x - 6 = 0$. $x = -2$ and $x = 3$
Objective 2

USING THE CALCULATOR BASED RANGER (CBR) WITH A TI-83+

Adapted from James P. Dildine (2007)

The CBR is a distance sensor that determines its location with respect to an object (it sends out sound waves then determines the time it takes for them to hit the object and bounce back, thus giving its location, or distance from the CBR).

SET-UP
First, make sure there are “good” batteries in the two devices (4 AA batteries for the CBR and 4 AAA batteries for the TI-83+) and then connect the two devices using the TI link cable. Important Note: Make sure that the cables are all the way in each device (you should hear an audible “click”, indicating they are in securely).

The CBL/CBR application allows real-time graphing, so we prefer it for this activity.
To begin, turn the calculator ON by pressing the ON key located at the bottom left. Then press the APPS key to get the following menu.

From this menu choose CBL/CBR (as circled) by using the arrow keys (or the number it is assigned to, in this case “2”) to highlight the CBL/CBR option and pressing ENTER.
You will then see the following screen, so hit any key and you’ll get the next screen.

Then choose the Number 3:RANGER option by arrowing down or pressing the “3” key.
You will then see a menu that tells you to press ENTER.
Objective 2

Press ENTER and you’ll view the following menu.

```
MAIN MENU
1: SETUP/SAMPLE
2: SET DEFAULTS
3: APPLICATIONS
4: PLOT MENU
5: TOOLS
6: QUIT
```

First, choose to create our own graphs with the CBR:
We choose option 1: SETUP/SAMPLE and press ENTER and we are presented with a menu.
We want to set the following options (pressing ENTER allows you to choose from each possible option):

- **REAL TIME**: YES – This means we can see our graph as it is made
- **TIME(S)**: 15 – Real time mode restricts us to 15 seconds
- **DISPLAY**: DIST – This changes the type of graph we see (DIST v. TIME, ACCEL v. TIME, and VELOCITY v. TIME) the graph we want is DIST
- **BEGIN ON**: [ENTER] – We can choose the CBR trigger option but [ENTER] is easier.
- **SMOOTHING**: MEDIUM – Presents our graph in a more readable manner (taking out extraneous points)
- **UNITS**: FEET – Distance will be measured in feet

Then we arrow up to START NOW and hit ENTER

To get the next screen, follow the directions presented.

```
POINT CBR AT TARGET

TO START PRESS [ENTER] ON TI83P
```

Your target will be a wall (with you are holding the CBR). Point the CBR at a wall and walk back and forth and your graph should look like the following graph. Think about what the graph looks like and what it means.

```
<table>
<thead>
<tr>
<th>TIME (S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
</tr>
<tr>
<td>0.5</td>
</tr>
<tr>
<td>0.75</td>
</tr>
<tr>
<td>1.0</td>
</tr>
</tbody>
</table>

X=1.29  Y=.916
```
Objective 2

After creating your first graph press the [ENTER] key and you’ll be presented with the following options.

1: SHOW PLOT — Allows you to view the graph again
2: SELECT DOMAIN — Allows you to analyze a specific portion of the graph
3: REPEAT SAMPLE — Allows you to create a new graph
4: MAIN MENU — Brings you back to the main/setup menu
5: QUIT — Quits the CBR program

Additionally we can use built in programs to explore graphs in particular, the MATCH-THE-GRAPH activities. When you are at the main menu:

Choose 3: APPLICATIONS

And you’ll get a menu like this

Choose your measurement units (feet).
Objective 2

Then you'll see this menu.

Choose 1: DIST MATCH. Then you'll see something similar to this (or another type of graph).

The idea is to try and use the CBR and your motion to create a graph similar to the graph presented to you. Look at the slope of the line over the time and try to create a graph that looks like the one presented. You might have to walk fast, slow, or stop completely to create the different stages of your graph. See how close you can get to matching the graph!

Sample Situation:
My attempt at the graph shown above is shown here (the dotted plot is my graph). I recognized that I needed to start about 2 feet from the wall. I then saw that I should move away from the wall, not too quickly, then stay still for about 5 seconds, and then move toward the wall, again not too quickly, but quicker than I moved away from the wall.
Objective 2

ACTIVITIES TO TRY

After you master the Match-the-Graph activities you should attempt to create and match your own graphs. Work with a partner to try to create each of the following.

Some graphs you may want to try are presented here (with space to describe how you created them or cannot create them) making note of speed and location from the wall or sensor:

1. ![Graph 1]
   Explanation:

2. ![Graph 2]
   Explanation:

3. ![Graph 3]
   Explanation:

Then, use the following graph to draw your own graph and have a partner create it with the CBR.
Slide 7

What is the domain for the set of coordinates \{(-2,1), (0,0), (2, 1)\}?

Slide 8

\{-2, 0, 2\}

Slide 9

What is the total for dinner if the check was $43.18 and you left a 20% tip?
Slide 10

Slide 11

Simplify: \((2x + 1)^2 - 4(x - 3)^2\)

Slide 12

\[28x - 35\]
Slide 13

Which property is illustrated by the equation \((5x + 3) + 2x = 5x + (3 + 2x)\)?

Slide 14

Associative Property of Addition

Slide 15

What is the perimeter of a rectangle with sides of \(x\) and \(x + 2\)?
Slide 16

Slide 17
What is the next item in the pattern $2x, 4x^2, 8x^3, \ldots$?

Slide 18

16x^3
Slide 19

What is the area of a rectangle with sides \((x - 1)\) and \((x + 5)\)?

Slide 20

\[x^2 + 4x - 5\]

Slide 21

For the function \(y = 3x + 4\), what is the value of \(y\) when \(x\) is 5?
APPENDIX C

OBJECTIVE 6 MATERIALS
Content Notes
Graphing on the Coordinate Plane

**TEKS:**
8.7(D): The student uses geometry to model and describe the physical world. The student is expected to locate and name points on a coordinate plane using ordered pairs of rational numbers.

**Vocabulary:**
- coordinate plane
- origin
- quadrant
- x-axis
- x-coordinate
- y-axis
- y-coordinate

The coordinate plane is formed by the perpendicular intersection of two number lines, the x-axis and the y-axis. These two lines divide the plane into four quadrants. The first quadrant contains positive x and y values. The quadrants then move counter-clockwise. The x-coordinate is the first number listed in an ordered pair and the y-coordinate is the second.

To plot points on the coordinate plane, begin at the origin, the point (0,0). This point is the intersection of the x-axis and the y-axis. The x-coordinate tells you how many units to move left or right. The y-coordinate tells you how many units to move up or down. Moving right or up are positive movements while moving left or down are negative.
**Example 1:**
A (4, -1)
The x-coordinate is 4 and the y-coordinate is -1. You start at the origin and move 4 units right and 1 unit down. This point is located in Quadrant IV.

**Example 2:**
B (5, 0)
The x-coordinate is 5 and the y-coordinate is 0. You start at the origin and move 5 units right and 0 units down. This point is on the x-axis so it does not have a quadrant.

**Example 3:**

Find the coordinates of points A, B, and C.

- **A (-3, 6)** Start at the origin and move 3 units left and 6 units up. Point A is in Quadrant II.

- **B (6, -3)** Start at the origin and move 6 units right and 3 units down. Point B is in Quadrant IV.

- **C (2, 0)** Start at the origin and move 2 units right and no units up or down. Since point C is located on the x-axis, it is not in any of the quadrants.
Objective 6

Try it with a partner!!

1. Graph the ordered pairs E(-1, 6), F(0, -5), G(-2, -3) on the coordinate grid.

2. Write the coordinates for points A, B, C, and D.
   a. A ( , )
   b. B ( , )
   c. C ( , )
   d. D ( , )

3. Which two coordinates have the same x-coordinate?

4. Which two coordinates have the same y-coordinate?
Dilations and Scale Factor

**TEKS:**
8.6(A): The student uses transformational geometry to develop spatial sense. The student is expected to generate similar figures using dilations including enlargements and reductions.

**Vocabulary:**
center of dilation
dilation
scale factor

A dilation is a transformation that enlarges or reduces a figure. The ratio that is used to enlarge or reduce the figure is called the scale factor. Dilations produce similar figures.

**Example 1:**

Start with square ABCD. Each side of the square is 1 unit.
Use a scale factor of 5. That means that each side of the new square will be 5 times as large. We will use the origin as the center of dilation so to make it 5 times as large, take each coordinate and multiply by 5.

\[
\begin{align*}
A (-1, 1) & \quad \times 5 = A' (-5, 5) \\
B (1, 1) & \quad \times 5 = B' (5, 5) \\
C (1, -1) & \quad \times 5 = C' (5, -5) \\
D (-1, -1) & \quad \times 5 = D' (-5, -5)
\end{align*}
\]

Square A'B'C'D' has side lengths of 5 and each of the sides are 5 times as large as the original.
Example 2:
Find the scale factor used to go from triangle ABC to triangle A'B'C'.

Triangle A'B'C' is smaller than triangle ABC so the scale factor must be less than 1. To find the scale factor find the lengths of two similar sides.
A'B' = 6 units
AB = 12 units
The scale factor is \( \frac{A'B'}{AB} \rightarrow \frac{6}{12} = \frac{1}{2} \)
Transformations

**TEKS:**
8.6(B): The student uses transformational geometry to develop spatial sense. The student is expected to graph dilations, reflections, and translations on a coordinate plane.

**Vocabulary:**
center of rotation
reflection
rotation
transformation
translation

A transformation is a change in a figure’s position or size. A translation slides a figure along a line without turning. A rotation turns a figure around a point called the center of rotation. A reflection flips a figure across a line. The resulting images from a translation, rotation, or reflection are congruent.

**Example 1:**
Graph the points A (-1, 2), B (4, 7), and C (7, 4) to make triangle ABC. Use the translation \((x - 2, y - 2)\) to make triangle A’B’C’. The x-coordinate moves the point to the left or right and the y-coordinate moves the point up or down. This translation tells you to move 2 units to the left \((x - 2)\) and 2 units down \((y - 2)\).

What are the coordinates for:

A’? _______________
B’? _______________
C’? _______________

What do you notice about the difference in the coordinates? What operation do you perform for translations?

______________________________________________

______________________________________________

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Example 2:
Using the same points for triangle ABC, rotate the triangle 90° counter-clockwise around the origin.

Then try 180°, 270° and 360°.

What happens to the coordinates during these counter-clockwise rotations?

90° (x, y) → (-y, x)
180° (x, y) → (____  ____)
270° (x, y) → (____  ____)
360° (x, y) → (____  ____)

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**Example 3:**

Using the same points for triangle ABC, reflect across the x-axis and then across the y-axis.
What happens to the coordinates during these reflections?

**X-Axis Reflection**

![Triangle ABC reflected across the x-axis](image)

Across the x-axis \((x, y) \rightarrow (\_, \_\_)\)

**Y-Axis Reflection**

![Triangle ABC reflected across the y-axis](image)

Across the y-axis \((x, y) \rightarrow (\_, \_\_)\)
**Content Notes Key**

**Graphing on the Coordinate Plane**

**TEKS:**
8.7(D): The student uses geometry to model and describe the physical world. The student is expected to locate and name points on a coordinate plane using ordered pairs of rational numbers.

**Vocabulary:**
coordinate plane
x-axis
y-axis
quadrant
x-coordinate
y-coordinate
origin

The coordinate plane is formed by the perpendicular intersection of two number lines, the x-axis and the y-axis. These two lines divide the plane into four quadrants. The first quadrant contains positive x and y values. The quadrants then move counter-clockwise. The x-coordinate is the first number listed in an ordered pair and the y-coordinate is the second.

To plot points on the coordinate plane, begin at the origin, the point (0,0). This point is the intersection of the x-axis and the y-axis. The x-coordinate tells you how many units to move left or right. The y-coordinate tells you how many units to move up or down. Moving right or up are positive movements while moving left or down are negative.
Objective 6

**Example 1:**
A (4, -1)
The x-coordinate is 4 and the y-coordinate is -1. You start at the origin and move 4 units right and 1 unit down. This point is located in Quadrant IV.

**Example 2:**
B (5, 0)
The x-coordinate is 5 and the y-coordinate is 0. You start at the origin and move 5 units right and 0 units down. This point is on the x-axis so it does not have a quadrant.

**Example 3:**
Find the coordinates of points A, B, and C.

- A (-3, 6) Start at the origin and move 3 units left and 6 units up. Point A is in Quadrant II.

- B (6, -3) Start at the origin and move 6 units right and 3 units down. Point B is in Quadrant IV.

- C (2, 0) Start at the origin and move 2 units right and no units up or down. Since point C is located on the x-axis, it is not in any of the quadrants.
1. Graph the ordered pairs E(-1, 6), F(0, -5), G(-2, -3) on the coordinate grid.

2. Write the coordinates for points A, B, C, and D.
   a. A \((3, 5)\)
   b. B \((3, 1)\)
   c. C \((-4, 5)\)
   d. D \((6, -8)\)

3. Which two coordinates have the same x-coordinate? A & B

4. Which two coordinates have the same y-coordinate? A & C
Dilations and Scale Factor

**TEKS:**
8.6(A): The student uses transformational geometry to develop spatial sense. The student is expected to generate similar figures using dilations including enlargements and reductions.

**Vocabulary:**
center of dilation
dilation
scale factor

A dilation is a transformation that enlarges or reduces a figure. The ratio that is used to enlarge or reduce the figure is called the scale factor. Dilations produce similar figures.

**Example 1:**

Start with square ABCD. Each side of the square is 1 unit.
Use a scale factor of 5. That means that each side of the new square will be 5 times as large. We will use the origin as the center of dilation so to make it 5 times as large, take each coordinate and multiply by 5.

\[
\begin{align*}
A (-1, 1) & \times 5 = A' (-5, 5) \\
B (1, 1) & \times 5 = B' (5, 5) \\
C (1, -1) & \times 5 = C' (5, -5) \\
D (-1, -1) & \times 5 = D' (-5, -5)
\end{align*}
\]

Square A'B'C'D' has side lengths of 5 and the lengths of the sides are 5 times as large as the original.
Example 2:
Find the scale factor used to go from triangle ABC to triangle A'B'C'.

Triangle A'B'C' is smaller than triangle ABC so the scale factor must be less than 1. To find the scale factor find the lengths of two similar sides.
A'B' = 6 units
AB = 12 units
The scale factor is A'B'/AB \rightarrow \frac{6}{12} = \frac{1}{2}
Transformations

TEKS:
8.6(B): The student uses transformational geometry to develop spatial sense. The student is expected to graph dilations, reflections, and translations on a coordinate plane.

Vocabulary:
center of rotation
reflection
rotation
transformation
translation

A transformation is a change in a figure’s position or size. A translation slides a figure along a line without turning. A rotation turns a figure around a point called the center of rotation. A reflection flips a figure across a line. The resulting images from a translation, rotation, or reflection are congruent.

Example 1:
Graph the points A (-1, 2), B (4, 7), and C (7, 4) to make triangle ABC. Use the translation (x – 2, y – 2) to make triangle A’B’C’. The x-coordinate moves the point to the left or right and the y-coordinate moves the point up or down. This translation tells you to move 2 units to the left (x – 2) and 2 units down (y – 2).

What are the coordinates for:

A’? (-3, 0)
B’? (2, 5)
C’? (5, 2)

What do you notice about the difference in the coordinates? What operation do you perform for translations? **The x-coordinate decreases by 2 and the y-coordinate decreases by 2. You add or subtract when performing translations.**
Objective 6

Example 2:
Using the same points for triangle ABC, rotate the triangle $90^\circ$ counter-clockwise around the origin.

Then try $180^\circ$, $270^\circ$ and $360^\circ$.

What happens to the coordinates during these counter-clockwise rotations?

$90^\circ (x, y) \rightarrow (-y, x)$
$180^\circ (x, y) \rightarrow (y, x)$
$270^\circ (x, y) \rightarrow (-x, y)$
$360^\circ (x, y) \rightarrow (x, y)$
Example 3:
Using the same points for triangle ABC, reflect across the x-axis and then across the y-axis.
What happens to the coordinates during these reflections?

X-Axis Reflection

Across the x-axis \((x, y) \rightarrow (x, -y)\)

Y-Axis Reflection

Across the y-axis \((x, y) \rightarrow (-x, y)\)
Student Activity
Using Geometer’s Sketchpad

Dilations

Directions:

1. Open a new sketch.

2. Go to Graph → Grid Form → Square Grid to turn on the coordinate plane.

3. Go to Graph → Plot Points.

4. Plot three points on your graph.

5. Connect your three points using the Line Segment Tool.

6. Label your points A, B, and C using the Text Tool.

7. Select your three points and line segments together.

8. Dilate your triangle by going to Transform → Dilate → Fixed Ratio → \( \frac{1}{2} \).

9. Compare your original coordinates to your translated coordinates. What happens to the coordinates of your triangle after performing this dilation?

10. Repeat step 8 but with a fixed ratio of 2.

11. What happens to the coordinates of your triangle after performing this dilation?

12. When does a dilation make a figure larger?

13. When does a dilation make a figure smaller?
Translations

Directions:

1. Open a new sketch.
2. Go to Graph $\rightarrow$ Grid Form $\rightarrow$ Square Grid to turn on the coordinate plane.
3. Go to Graph $\rightarrow$ Plot Points.
4. Plot three points on your graph.
5. Connect your three points using the Line Segment Tool.
6. Label your points A, B, and C using the Text Tool.
7. Select your three points and line segments together.
8. Translate your triangle by going to Transform $\rightarrow$ Translate $\rightarrow$ Translation Vector

Rectangular.

9. Horizontal (change in the x-coordinate) $\rightarrow$ 3.

10. Vertical (change in the y-coordinate) $\rightarrow$ 3.

14. What happens to the coordinates of your triangle after performing this translation?

15. Repeat step 8 but with a different translation vector. Write your translation here.

16. What happens to the coordinates of your triangle after performing this translation?

17. What happens to the coordinates of your triangle when you perform translations?

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Rotations

Directions:

1. Open a new sketch.

2. Go to Graph → Grid Form → Square Grid to turn on the coordinate plane.

3. Go to Graph → Plot Points.

4. Plot three points on your graph.

5. Connect your three points using the Line Segment Tool.

6. Label your points A, B, and C using the Text Tool.

7. Select your three points and line segments together.

8. Rotate your triangle by going to Transform → Rotate → Fixed Angle → 90° (about the origin).

   What happens to the coordinates of your triangle after performing this rotation?

   90° (x, y) becomes ( , )

9. Repeat step 8 but with a 180°, 270° and 360° rotation.

   What happens to the coordinates of your triangle after each rotation?

   180° (x, y) → ( , )

   270° (x, y) → ( , )

   360° (x, y) → ( , )
Reflections

Directions:

1. Open a new sketch.
2. Go to Graph → Grid Form → Square Grid to turn on the coordinate plane.
3. Go to Graph → Plot Points.
4. Plot three points on your graph.
5. Connect your three points using the Line Segment Tool.
6. Label your points A, B, and C using the Text Tool.
7. Select the x-axis.
8. Go to Transform → Mark Mirror. This will set the x-axis as your reflection axis.
9. De-select the x-axis and select your three points and line segments together.
10. Reflect your triangle over the x-axis by going to Transform → Reflect.
11. What happened to the coordinates of your triangle after this reflection?

12. De-select everything on your sketch. Select the y-axis and go to Transform → Mark Mirror. This will set the y-axis as your axis for reflection.
13. Select your original three points and the line segments together.
14. Reflect your triangle over the y-axis by going to Transform → Reflect.
15. What happened to the coordinates of your triangle after this reflection?

16. In general, what happens to coordinates when you reflect over the x and y-axis?

x-axis \((x, y) \rightarrow (\_, \_\_\_\_\_\_\_\_\_\_\_\_\_\_)\)

y-axis \((x, y) \rightarrow (\_, \_\_\_\_\_\_\_\_\_\_\_\_\_)\)
Objective 6 – Adapted from Mark E. Damon (2002)

Slide 1

Welcome to...

A Game of X's and O's

Slide 2

Another

Presentation
© 2000 - All rights Reserved
Questions created by
Jennifer Jackson

Slide 3

Sand/Beaches
Objective 6 – Adapted from Mark E. Damon (2002)

Slide 7

A rectangle with dimensions 2 ft by 5 ft is dilated with a scale factor of 3. What are the new dimensions?

Slide 8

6 ft by 15 ft

Slide 9

What quadrant is the star in?
Slide 10

Triangle ABC is dilated with a scale factor of \( \frac{3}{2} \).

The coordinate of \( A \) is \((10, 8)\). What is \( A' \)?

Slide 11

Slide 12

(5, 4)
Objective 6 – Adapted from Mark E. Damon (2002)

Slide 13

What does the coordinate \((x, y)\) become when you reflect over the x-axis?

Slide 14

\((x, -y)\)

Slide 15

What operation is performed on coordinates that are dilated?
Slide 16

Multiplication

Slide 17

What happens to the coordinate (-1, 8) under the translation (x + 4, y - 4)?

Slide 18

(3, 4)
Slide 19

What does it mean if similar figures have a scale factor of 3?

Slide 20

3 times as big

Slide 21

Will a scale factor of \( \frac{1}{4} \) make a figure smaller or larger?
APPENDIX D

OBJECTIVE 8 MATERIALS
Content Notes
Lateral and Total Surface Area of
Three-Dimensional Figures

TEKS:
8.8(A): The student uses procedures to determine measures of three-dimensional figures. The student is expected to find lateral and total surface area of prisms, pyramids, and cylinders using concrete models and nets (two-dimensional models).

8.8(C): The student uses procedures to determine measures of three-dimensional figures. The student is expected to estimate measurements and use formulas to solve application problems involving lateral and total surface area and volume.

Vocabulary:
cone
cylinder
lateral area
net
prism
pyramid
sphere
surface area

A net is an arrangement of two-dimensional figures that can be folded to form a three-dimensional figure. Basically, it is the unfolded version of a three-dimensional figure. Nets can be helpful to find the area of three-dimensional figures. There are two different types of surface area for most three-dimensional figures, lateral area and total surface area. Lateral area is the area of all of the lateral faces of a figure. These are all of the planar faces of a figure that are not bases. Total surface area is the lateral area plus the area of the base(s). Since a sphere does not have a base, there is only one type of surface area, total surface area, for a sphere.
Example 1:
What is the shape of the base of a rectangular prism? _______________________

Lateral area is the area of all of the faces other than the two bases. Find the area of all of the rectangles other than the two bases and add them together. This will give you the lateral area. To find the total surface area, add the lateral area to the area of the two bases.

Use the net provided to find the lateral and total surface area of the rectangular prism. Use the scale on the axis for measurements.

Lateral area: _______________________

Total surface area: _______________________
Objective 8

**Example 2:**
What is the shape of the base of a cylinder? ________________
What is the shape of the lateral surface of a cylinder? ________________

To find the lateral area of the net of a cylinder, just find the area of the rectangle that forms the lateral surface. To find the total surface area, add the lateral area to the areas of the two circle bases.

Use the net provided to find the lateral and total surface area of the cylinder. Measure the dimensions to the nearest tenth of a centimeter.

What is the radius of the base of this cylinder? __________ cm
What is the height of this cylinder? __________ cm
What is the lateral area? __________ cm²
What is the total surface area? __________ cm²
Example 3:
Measure the dimensions of the square pyramid to the nearest tenth of a centimeter. Find the lateral and total surface area.

Lateral area: __________ cm²

Total surface area: __________ cm²
Objective 8

**Volume of Three-Dimensional Figures**

**TEKS:**
8.8(B): The student uses procedures to determine measures of three-dimensional figures. The student is expected to connect models of prisms, cylinders, pyramids, spheres, and cones to formulas for volume of these objects.

8.8(C): The student uses procedures to determine measures of three-dimensional figures. The student is expected to estimate measurements and use formulas to solve application problems involving lateral and total surface area and volume.

**Vocabulary:**
volume

Volume is how many cubic units a figure can hold or the amount of space inside a three-dimensional figure.

**Example 1:**
Find the volume of the rectangular prism. Use the dimensions given by the scale on the axis.

What is the area of the base (B)?

What is the height of the prism (h)?

What is the volume of this rectangular prism?
Objective 8

**Example 2:**
Volume of a cylinder can be found by using the same formula as a prism, \( V = Bh \). The base is a circle and the formula for area of a circle is \( A = \pi r^2 \). So the formula for volume of a cylinder is \( V = \pi r^2 h \).

Measure the dimensions of the cylinder in centimeters.

What is the radius of the base of this cylinder? _______ cm
What is the height of this cylinder? _______ cm
What is the volume of this cylinder? _______ cm³
Objective 8

**Pythagorean Theorem**

**TEKS:**
8.9(A): The student uses indirect measurement to solve problems. The student is expected to use the Pythagorean Theorem to solve real-life problems.

**Vocabulary:**
hypotenuse
leg
Pythagorean Theorem

The Pythagorean Theorem is useful to find a missing side of a right triangle when the other two sides are known. The Pythagorean Theorem states that in any right triangle, the sum of the squares of the lengths of the two legs is equal to the square of the hypotenuse. \((a^2 + b^2 = c^2)\)
Right triangles are found everywhere and used in daily life.

**Example 1:**
The ladder to a playground slide and the slide itself form a right triangle with the ground. If the ladder is 6 feet tall and the distance from the foot of the ladder to the end of the slide is 8 ft, how long is the slide?

Draw and label a picture.
Then, set up the problem and solve.

The unknown side is the hypotenuse, or the c.
\[ a^2 + b^2 = c^2 \]
\[ 6^2 + 8^2 = c^2 \]
\[ 36 + 64 = c^2 \]
\[ 100 = c^2 \]
\[ \sqrt{100} = c \]
\[ 10 = c \]
The length of the slide is 10 feet.
Example 2:
Use the grid to solve the following problems.

There is a serious car accident. The closest fire station to this accident is Fire Station #3. The police have dispatched an ambulance and a helicopter to the scene. The ambulance must follow the streets, but the helicopter flies from the fire station straight to the site.

1. How many miles west of the fire station is the accident? ________________
   (Draw this horizontal line on the grid)

2. How many miles north of the fire station is the accident? ________________
   (Draw this vertical line on the grid)

3. What is the total number of miles the ambulance must travel? ______________

4. Draw a straight line from the fire station to the accident. What theorem could you use to find this distance? ____________________________

5. Use this theorem to find the number of miles the helicopter must travel. ____________

6. How many more miles does the ambulance have to travel to the accident than the helicopter does? ____________________________
Proportional Relationships

**TEKS:**
8.9(B): The student uses indirect measurement to solve problems. The student is expected to use proportional relationships in similar two-dimensional figures or similar three-dimensional figures to find missing measurements.

**Vocabulary:**
- indirect measurement
- proportion
- scale factor
- similar figures

Similar figures have the same shape, but not necessarily the same size. When shapes are similar, their angles are congruent and the lengths of corresponding sides are proportional. To find the similarity ratio of two similar figures, you must set up a proportion of corresponding sides.

**Example 1:**
Given: \( \triangle ABC \sim \triangle DEF \)

![Diagram of triangles ABC and DEF]

Given that \( \triangle ABC \sim \triangle DEF \), find \( x \) and \( y \).

Which two corresponding sides do you know the side lengths for? ________________

You can set this up as a proportion. To write this as a fraction, put the side that you know from \( \triangle ABC \) over the corresponding side that you know from \( \triangle DEF \).

\[
\frac{AB}{DE} = \frac{EF}{x} \quad \text{which reduces to} \quad \frac{2}{1}
\]

Now to solve for \( x \) and \( y \), you use this proportion and the sides that correspond to \( x \) and \( y \) on the other triangle. Sides from \( \triangle ABC \) are on the top and sides from \( \triangle DEF \) are on the bottom when you set up your proportion.
Objective 8

To solve for $x$:
\[
\frac{2}{1} = \frac{x}{5}
\]
Cross multiply to solve for $x$.
\[x = 10\]
So, $AC = 10$.

To solve for $y$:
\[
\frac{2}{1} = \frac{8}{y}
\]
Cross multiply to solve for $y$.
\[2y = 8\]
Divide by 2 to get $y$ by itself.
\[y = 4\]
So, $EF = 4$.

Example 2:

Given that $\triangle ABC \sim \triangle DEC$, find $x$, $y$ and $z$.

\[x = \underline{\phantom{000}}\]  \hspace{1cm} (Hint: Use Pythagorean Theorem)

\[y = \underline{\phantom{000}}\]  \hspace{1cm} (Hint: Use a proportion)

\[z = \underline{\phantom{000}}\]
Scaling Two and Three-Dimensional Figures

**TEKS:**

8.10(A): The student describes how changes in dimensions affect linear, area, and volume measures. The student is expected to describe the resulting effects on perimeter and area when the dimensions of a shape are changed proportionally.

8.10(B): The student describes how changes in dimensions affect linear, area, and volume measures. The student is expected to describe the resulting effect on volume when dimensions of a solid are changed proportionally.

**Vocabulary:**

Similar three-dimensional figures have special properties when you change the dimensions proportionally. Area is measured in square units, so if you multiply the dimensions of a solid by $n$, the surface area will change by $n^2$. For example, if you have a cube with sides of 2, the total surface area is $(SA = 6s^2)\ 6*2^2 = 24 \text{ units}^2$. If you double all of the dimensions of the cube, the sides become 4. The new surface area is $6*4^2 = 96 \text{ units}^2$. If you compare the new area with the original (96/24), the surface area is 4 times ($2^2$) larger.

Similarly, volume has the same type of relationship. Volume is measured in cubic units. If you multiply the dimensions of a solid by $n$, the volume will change by $n^3$. Using the same example with a cube with sides of 2, the volume is $(V = Bh)\ 2^2*2 = 8 \text{ units}^3$. If we increase all of the dimensions by a factor of 2, the new volume would be $4^2*4 = 64 \text{ units}^3$. If you compare the new volume with the original (64/8), the volume is 8 times ($2^3$) larger.

This pattern will continue anytime you multiply the dimensions of a solid by a given constant ($n$). Surface area will change by a factor of $n^2$ and volume will change by a factor of $n^3$.
Objective 8

**Example 1:**
A cylindrical gas tank is going to be built at the local refinery. The president of the company would like to see a model of the tank before it is built. The actual tank is going to have a radius of 20 feet and a height of 40 feet. The president would like the model to be 1/10 the size of the actual tank.

What would be the radius of the model? ____________________
What would be the height of the model? ____________________
What is the formula for volume of a cylinder? ________________
What is the volume of the actual tank? _____________________
What is the volume of the model? ___________________________

How does the volume of the model compare to the volume of the actual tank? (In other words, how many times more space can be fit into the actual tank than the model?) ____________________________________________

**Example 2:**
A rectangular prism is doubled in size. What effect does this enlargement have on the surface area and volume?

Surface area: ______________________________
Volume: ________________________________
**Important Formulas**

<table>
<thead>
<tr>
<th>Perimeter</th>
<th>Rectangle</th>
<th>$P = 2l + 2w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circumference</td>
<td>Circle</td>
<td>$C = 2\pi r$ or $\pi d$</td>
</tr>
<tr>
<td>Area</td>
<td>Rectangle</td>
<td>$A = lw$</td>
</tr>
<tr>
<td></td>
<td>Triangle</td>
<td>$A = \frac{1}{2}bh$</td>
</tr>
<tr>
<td></td>
<td>Trapezoid</td>
<td>$A = \frac{1}{2}(b_1 + b_2)h$</td>
</tr>
<tr>
<td></td>
<td>Regular Polygon</td>
<td>$A = \frac{1}{2}aP$ (a = apothem, $P = $ perimeter)</td>
</tr>
<tr>
<td></td>
<td>Circle</td>
<td>$A = \pi r^2$</td>
</tr>
</tbody>
</table>

$P = $ Perimeter of the Base  
$B = $ Area of the Base

<table>
<thead>
<tr>
<th>Surface Area</th>
<th>Cube (total)</th>
<th>$SA = 6s^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Prism (lateral)</td>
<td>$LA = Ph$</td>
</tr>
<tr>
<td></td>
<td>Prism (total)</td>
<td>$SA = LA + 2B$</td>
</tr>
<tr>
<td></td>
<td>Pyramid (lateral)</td>
<td>$LA = \frac{1}{2}Pl$ (l = slant height)</td>
</tr>
<tr>
<td></td>
<td>Pyramid (total)</td>
<td>$SA = LA + B$</td>
</tr>
<tr>
<td></td>
<td>Cylinder (lateral)</td>
<td>$LA = 2\pi rh$</td>
</tr>
<tr>
<td></td>
<td>Cylinder (total)</td>
<td>$SA = LA + 2\pi r^2$</td>
</tr>
<tr>
<td></td>
<td>Cone (lateral)</td>
<td>$LA = \pi rl$</td>
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<tr>
<td></td>
<td>Cone (total)</td>
<td>$SA = LA + \pi r^2$</td>
</tr>
<tr>
<td></td>
<td>Sphere</td>
<td>$SA = 4\pi r^2$</td>
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</table>

<table>
<thead>
<tr>
<th>Volume</th>
<th>Prism or Cylinder</th>
<th>$V = Bh$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pyramid or Cone</td>
<td>$V = \frac{1}{3}Bh$</td>
</tr>
<tr>
<td></td>
<td>Sphere</td>
<td>$V = \frac{4}{3}\pi r^3$</td>
</tr>
</tbody>
</table>

**Pythagorean Theorem**

$$a^2 + b^2 = c^2$$
Objective 8

Content Notes Key
Lateral and Total Surface Area of Three-Dimensional Figures

**TEKS:**
8.8(A): The student uses procedures to determine measures of three-dimensional figures. The student is expected to find lateral and total surface area of prisms, pyramids, and cylinders using concrete models and nets (two-dimensional models).

8.8(C): The student uses procedures to determine measures of three-dimensional figures. The student is expected to estimate measurements and use formulas to solve application problems involving lateral and total surface area and volume.

**Vocabulary:**
cone
cylinder
lateral area
net
prism
pyramid
sphere
surface area

A net is an arrangement of two-dimensional figures that can be folded to form a three-dimensional figure. Basically, it is the unfolded version of a three-dimensional figure. Nets can be helpful to find the area of three-dimensional figures. There are two different types of surface area for most three-dimensional figures, lateral area and total surface area. Lateral area is the area of all of the lateral faces of a figure. These are all of the planar faces of a figure that are not bases. Total surface area is the lateral area plus the area of the base(s). Since a sphere does not have a base, there is only one type of surface area, total surface area, for a sphere.
Objective 8

**Example 1:**
What is the shape of the base of a rectangular prism? *rectangle*

Lateral area is the area of all of the faces other than the two bases. Find the area of all of the rectangles other than the two bases and add them together. This will give you the lateral area. To find the total surface area, add the lateral area to the area of the two bases.

![Diagram of a rectangular prism net]

Use the net provided to find the lateral and total surface area of the rectangular prism. Use the scale on the axis for measurements.

Lateral area: \(16 + 16 + 40 + 40 = 112 \text{ units}^2\)

Total surface area: \(112 + 10 + 10 = 132 \text{ units}^2\)
Example 2:
What is the shape of the base of a cylinder? circle
What is the shape of the lateral surface of a cylinder? rectangle

To find the lateral area of the net of a cylinder, just find the area of the rectangle that forms the lateral surface. To find the total surface area, add the lateral area to the areas of the two circle bases.

Use the net provided to find the lateral and total surface area of the cylinder. Measure the dimensions to the nearest tenth of a centimeter.

What is the radius of the base of this cylinder? 1.8 cm
What is the height of this cylinder? 7.5 cm
What is the lateral area? $7.5 \times 11.2 = 84$ cm$^2$
What is the total surface area? $84 + 2\pi(1.8)^2 = 104.4$ cm$^2$
Example 3:
Measure the dimensions of the square pyramid to the nearest tenth of a centimeter.
Find the lateral and total surface area.

Lateral area: \( \frac{1}{2} (6.3)(4) \times 4 = 50.4 \text{ cm}^2 \)

Total surface area: \( 50.4 + (6.3)(6.3) = 90.1 \text{ cm}^2 \)
Volume of Three-Dimensional Figures

**TEKS:**
8.8(B): The student uses procedures to determine measures of three-dimensional figures. The student is expected to connect models of prisms, cylinders, pyramids, spheres, and cones to formulas for volume of these objects.

8.8(C): The student uses procedures to determine measures of three-dimensional figures. The student is expected to estimate measurements and use formulas to solve application problems involving lateral and total surface area and volume.

**Vocabulary:**
volume

Volume is how many cubic units a figure can hold or the amount of space inside a three-dimensional figure.

**Example 1:**
Find the volume of the rectangular prism. Use the dimensions given by the scale on the axis.

What is the area of the base (B)? $2 \times 3 = 6 \text{ units}^2$

What is the height of the prism (h)? $8 \text{ units}$

What is the volume of this rectangular prism? $6 \times 8 = 48 \text{ units}^3$
**Example 2:**
Volume of a cylinder can be found by using the same formula as a prism, \( V = Bh \). The base is a circle and the formula for area of a circle is \( A = \pi r^2 \). So the formula for volume of a cylinder is \( V = \pi r^2 h \).

Measure the dimensions of the cylinder in centimeters.

What is the radius of the base of this cylinder? \( 1.8 \) cm
What is the height of this cylinder? \( 7.5 \) cm
What is the volume of this cylinder? \( \pi (1.8)^2 (7.5) = 76.3 \) cm\(^3\)
Objective 8

Pythagorean Theorem

**TEKS:**
8.9(A): The student uses indirect measurement to solve problems. The student is expected to use the Pythagorean Theorem to solve real-life problems.

**Vocabulary:**
hypotenuse
leg
Pythagorean Theorem

The Pythagorean Theorem is useful to find a missing side of a right triangle when the other two sides are known. The Pythagorean Theorem states that in any right triangle, the sum of the squares of the lengths of the two legs is equal to the square of the hypotenuse. \(a^2 + b^2 = c^2\)
Right triangles are found everywhere and used in daily life.

**Example 1:**
The ladder to a playground slide and the slide itself form a right triangle with the ground. If the ladder is 6 feet tall and the distance from the foot of the ladder to the end of the slide is 8 ft, how long is the slide?

Draw and label a picture.
Then, set up the problem and solve.

The unknown side is the hypotenuse, or the c.
\[
a^2 + b^2 = c^2 \\
6^2 + 8^2 = c^2 \\
36 + 64 = c^2 \\
100 = c^2 \\
\sqrt{100} = c \\
10 = c
\]
The length of the slide is 10 feet.
Example 2:
Use the grid to solve the following problems.

There is a serious car accident. The closest fire station to this accident is Fire Station #3. The police have dispatched an ambulance and a helicopter to the scene. The ambulance must follow the streets, but the helicopter flies from the fire station straight to the site.

1. How many miles west of the fire station is the accident? 6 miles
(Draw this horizontal line on the grid)

2. How many miles north of the fire station is the accident? 7 miles
(Draw this vertical line on the grid)

3. What is the total number of miles the ambulance must travel? 13 miles

4. Draw a straight line from the fire station to the accident. What theorem could you use to find this distance? Pythagorean Theorem

5. Use this theorem to find the number of miles the helicopter must travel. 9.2 miles

6. How many more miles does the ambulance have to travel to the accident than the helicopter does? 13 − 9.2 = 3.8 miles more
Proportional Relationships

**TEKS:**
8.9(B): The student uses indirect measurement to solve problems. The student is expected to use proportional relationships in similar two-dimensional figures or similar three-dimensional figures to find missing measurements.

**Vocabulary:**
indirect measurement
proportion
scale factor
similar figures

Similar figures have the same shape, but not necessarily the same size. When shapes are similar, their angles are congruent and the lengths of corresponding sides are proportional. To find the similarity ratio of two similar figures, you must set up a proportion of corresponding sides.

**Example 1:**
Given: $\triangle ABC \sim \triangle DEF$

Given that $\triangle ABC \sim \triangle DEF$, find $x$ and $y$.

Which two corresponding sides do you know the side lengths for? $AB$ & $DE$
You can set this up as a proportion. To write this as a fraction, put the side that you know from $\triangle ABC$ over the corresponding side that you know from $\triangle DEF$.

\[
\frac{AB}{DE} = \frac{EF}{3} \quad \frac{6}{3} \text{ which reduces to } \frac{2}{1}
\]

Now to solve for $x$ and $y$, you use this proportion and the sides that correspond to $x$ and $y$ on the other triangle. Sides from $\triangle ABC$ are on the top and sides from $\triangle DEF$ are on the bottom when you set up your proportion.
Objective 8

To solve for $x$:
\[
\frac{2}{1} = \frac{x}{5}
\]
Cross multiply to solve for $x$.
\[x = 10\]
So, $AC = 10$.

To solve for $y$:
\[
\frac{2}{1} = \frac{8}{y}
\]
Cross multiply to solve for $y$.
\[2y = 8\]
Divide by 2 to get $y$ by itself.
\[y = 4\]
So, $EF = 4$.

Example 2:

Given that $\triangle ABC \sim \triangle DEC$, find $x$, $y$ and $z$.

$x = 8$ (Hint: Use Pythagorean Theorem)

$y = 3.2$ (Hint: Use a proportion)

$z = 2.4$
Scaling Two and Three-Dimensional Figures

**TEKS:**
8.10(A): The student describes how changes in dimensions affect linear, area, and volume measures. The student is expected to describe the resulting effects on perimeter and area when the dimensions of a shape are changed proportionally.

8.10(B): The student describes how changes in dimensions affect linear, area, and volume measures. The student is expected to describe the resulting effect on volume when dimensions of a solid are changed proportionally.

**Vocabulary:**

Similar three-dimensional figures have special properties when you change the dimensions proportionally. Area is measured in square units, so if you multiply the dimensions of a solid by n, the surface area will change by \(n^2\). For example, if you have a cube with sides of 2, the total surface area is \((SA = 6s^2) = 6 \times 2^2 = 24\) units\(^2\). If you double all of the dimensions of the cube, the sides become 4. The new surface area is \(6 \times 4^2 = 96\) units\(^2\). If you compare the new area with the original (96/24), the surface area is 4 times \((2^2)\) larger.

Similarly, volume has the same type of relationship. Volume is measured in cubic units. If you multiply the dimensions of a solid by n, the volume will change by \(n^3\). Using the same example with a cube with sides of 2, the volume is \((V = Bh) = 2^2 \times 2 = 8\) units\(^3\). If we increase all of the dimensions by a factor of 2, the new volume would be \(4^2 \times 4 = 64\) units\(^3\). If you compare the new volume with the original (64/8), the volume is 8 times \((2^3)\) larger.

This pattern will continue anytime you multiply the dimensions of a solid by a given constant \((n)\). Surface area will change by a factor of \(n^2\) and volume will change by a factor of \(n^3\).
Objective 8

Example 1:
A cylindrical gas tank is going to be built at the local refinery. The president of the company would like to see a model of the tank before it is built. The actual tank is going to have a radius of 20 feet and a height of 40 feet. The president would like the model to be 1/10 the size of the actual tank.

What would be the radius of the model? \(2 \text{ feet}\)
What would be the height of the model? \(4 \text{ feet}\)
What is the formula for volume of a cylinder? \(V = \pi r^2 h\)
What is the volume of the actual tank? \(\pi(20)^2(40) = 50,265.5 \text{ ft}^3\)
What is the volume of the model? \(\pi(2)^2(4) = 50.3 \text{ ft}^3\)

How does the volume of the model compare to the volume of the actual tank? (In other words, how many times more space can be fit into the actual tank than the model?)

\[
50,265.5 \div 50.3 \approx 1000 = 10^3
\]

Example 2:
A rectangular prism is doubled in size. What effect does this enlargement have on the surface area and volume?

Surface area: \(2^2 = 4 \text{ times as large}\)
Volume: \(2^3 = 8 \text{ times as large}\)
## Important Formulas

<table>
<thead>
<tr>
<th>Perimeter</th>
<th>Rectangle</th>
<th>( P = 2l + 2w )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circumference</td>
<td>Circle</td>
<td>( C = 2\pi r ) or ( \pi d )</td>
</tr>
<tr>
<td>Area</td>
<td>Rectangle</td>
<td>( A = lw )</td>
</tr>
<tr>
<td></td>
<td>Triangle</td>
<td>( A = \frac{1}{2}bh )</td>
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<tr>
<td></td>
<td>Trapezoid</td>
<td>( A = \frac{1}{2}(b_1 + b_2)h )</td>
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<tr>
<td></td>
<td>Regular Polygon</td>
<td>( A = \frac{1}{2}aP ) (( a = ) apothem, ( P = ) perimeter)</td>
</tr>
<tr>
<td></td>
<td>Circle</td>
<td>( A = \pi r^2 )</td>
</tr>
</tbody>
</table>

\( P = \) Perimeter of the Base  
\( B = \) Area of the Base

<table>
<thead>
<tr>
<th>Surface Area</th>
<th>Cube (total)</th>
<th>( SA = 6s^2 )</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Prism (lateral)</td>
<td>( LA = Ph )</td>
</tr>
<tr>
<td></td>
<td>Prism (total)</td>
<td>( SA = LA + 2B )</td>
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<tr>
<td></td>
<td>Pyramid (lateral)</td>
<td>( LA = \frac{1}{2}Pl ) (( l = ) slant height)</td>
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<td></td>
<td>Pyramid (total)</td>
<td>( SA = LA + B )</td>
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<tr>
<td></td>
<td>Cylinder (lateral)</td>
<td>( LA = 2\pi rh )</td>
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<tr>
<td></td>
<td>Cylinder (total)</td>
<td>( SA = LA + 2\pi r^2 )</td>
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<tr>
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<td>Cone (lateral)</td>
<td>( LA = \pi rl )</td>
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<td>Cone (total)</td>
<td>( SA = LA + \pi r^2 )</td>
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<tr>
<td></td>
<td>Sphere</td>
<td>( SA = 4\pi r^2 )</td>
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</tbody>
</table>

<table>
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<tr>
<th>Volume</th>
<th>Prism or Cylinder</th>
<th>( V = Bh )</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Pyramid or Cone</td>
<td>( V = \frac{1}{3}Bh )</td>
</tr>
<tr>
<td></td>
<td>Sphere</td>
<td>( V = \frac{4}{3}\pi r^3 )</td>
</tr>
</tbody>
</table>

### Pythagorean Theorem

\( a^2 + b^2 = c^2 \)
Objective 8

**Student Activities**

**Surface Area and Volume of Real-World Objects**

Have students bring in different three-dimensional objects from home such as cereal boxes or different sized balls (soccer, baseball, etc.). Have students measure the dimensions of the objects that they brought and find surface area and volume of each one. Have students compare their items with other student’s items in the class to estimate surface area and volume for the different figures from their classmates.

<table>
<thead>
<tr>
<th>Object</th>
<th>Length</th>
<th>Width</th>
<th>Height</th>
<th>Surface Area</th>
<th>Volume</th>
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**Surface Area and Volume with Nets**

Use the nets on the separate pages to explore relationships. Measure all of the dimensions of each net in centimeters. Find the area of each individual shape of the net and add them all together. This is the total surface area for your figure. Then, cut out each net and fold to see what shape is formed. Compare the area that you found by adding each shape’s area to the area you get when you use the total surface area formula for each figure. Are they the same or different?
Objective 8 – Adapted from Mark E. Damon (2002)

Slide 1

Welcome to...

A Game of X's and O's

Slide 2

Another Presentation
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Questions created by Jennifer Jackson

Slide 3

SandyEM/Beach's
Slide 4

What happens to the area of a circle if the radius is doubled?

The area is 4 times larger
Objective 8 – Adapted from Mark E. Damon (2002)

Slide 7

The two rectangles are similar, find the value of x.

Slide 8

Slide 9

Find the volume of a cylindrical can with a radius of 4 cm and a height of 12 cm.
Objective 8 – Adapted from Mark E. Damon (2002)

Slide 10

Find the volume of ice cream in a cone with a radius of 2 cm and a height of 8 cm with a perfect semicircle top.

≈ 603 cm³

Slide 11

≈ 38 cm³
Find the hypotenuse of an isosceles right triangle with sides of 5 in.

\[ \approx 7.1 \text{ in} \]

What is the height and area of a rectangle with a width of 1 foot and a diagonal of \( \sqrt{10} \) feet.
Objective 8 – Adapted from Mark E. Damon (2002)

Slide 16

Height = 3 ft
Area = 3 ft²

Slide 17

The area of a triangle is 8 cm². The triangle is dilated with a scale factor of 2, what is the area of the new triangle?

Slide 18

32 cm²
Objective 8 – Adapted from Mark E. Damon (2002)

Slide 19

The volume of a triangle is 8 times larger than another triangle. What is the ratio of the radii of these two triangles?

Slide 20

Slide 21

What is the total surface area of a rectangular prism with base dimensions of 2 in and 4 in and a height of 5 in?
APPENDIX E

Cumulative Game

TEKS Contained by Objective

Important Vocabulary

Graph Paper
Welcome to

Who Wants to be a Millionaire

Another Presentation
Questions by Jennifer Jackson
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https://www.jenniferjackson.com
Cumulative Game – Adapted from Mark E. Damon (2002)

Slide 4

Slide 5

To what family of functions does 3x + 5y = 10 belong?

Slide 6
Slide 7

What is the set of all x-coordinates of a function?

Slide 8

Slide 9

What is a mathematical statement that contains at least one mathematical operation and at least two numbers and/or variables?
A graph shows the altitude of a hot air balloon as a function of the time since the balloon began to ascend. Which of the following would be the most appropriate domain for the graph?
Slide 13

What is the next term of the following sequence:
0, 1, -1, 2, -2, 3, ...

Slide 14

Congratulations!
You've Reached the $1,000 Milestone!

Slide 15

...
Slide 16

\[ g(x) = -3x^2 + 4x + 1 \]

Evaluate \( g(-3) \)

Slide 17

Slide 18

Simplify:

\[
\frac{1}{2}(8ab + 12b) - 2ab + 4a
\]
Slide 20

A triangle to dilated with a scale factor of 3? What will happen to the new triangle?

---

Slide 21
Slide 22

What is the volume of a cylinder with a diameter of 5 feet and a height of 10 feet?

A. 157.08 ft³  
B. 314.16 ft³  
C. 628.32 ft³  
D. 1256.64 ft³

Slide 23

Slide 24

When a power is divided by another power with the same base, which operation is performed on the exponents?

A. Addition  
B. Subtraction  
C. Multiplication  
D. Division

125
Slide 25

Congratulations!
You've Reached
the $32,000
Milestone!

Slide 26

Slide 27

What happens to the coordinate (-3, 4) under the translation (x + 4, y - 4)?
Cumulative Game – Adapted from Mark E. Damon (2002)

Slide 28

Slide 29

The area of a triangle is 5 in². A new triangle is formed with a scale factor of 3. What is the area of the new triangle?

Slide 30
Slide 31

Triangle ABC is made with A(12, 6), B(4, 4) and C(0,0). A new triangle is formed by dilating ABD with a scale factor of 1/3. What is the new coordinate of A?

Slide 32

Slide 33

Which of the following equations describes the quadratic parent function after it has been shifted up 5 units?
Cumulative Game – Adapted from Mark E. Damon (2002)

Slide 34

Slide 35
What does the coordinate (x, y) become when you reflect it over the y-axis?

Slide 36
YOU WIN $1 MILLION DOLLARS!
TEKS Contained in Each Objective

Taken from TEA (2006)

**Objective 2**

A.2(A)

- The student uses the properties and attributes of functions. The student is expected to identify and sketch the general forms of linear \( y = x \) and quadratic \( y = x^2 \) parent functions.

A.2(B)

- The student uses the properties and attributes of functions. The student is expected to identify mathematical domains and ranges and determine reasonable domain and range values for given situations, both continuous and discrete.

A.2(C)

- The student uses the properties and attributes of functions. The student is expected to interpret situations in terms of given graphs or creates situations that fit given graphs.

A.2(D)

- The student uses the properties and attributes of functions. The student is expected to collect and organize data, make and interpret scatterplots (including recognizing positive, negative and no correlation for data approximating linear situations), and model, predict, and make decisions and critical judgments in problem situations.

A.3(A)

- The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations. The student is expected to use symbols to represent unknowns and variables.

A.3(B)

- The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations. The student is expected to look for patterns and generalizations algebraically.

A.4(A)
• The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. The student is expected to find specific function values, simplify polynomial expressions, transform and solve equations, and factor as necessary in problem situations.

A.4(B)

• The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. The student is expected to use the commutative, associative, and distributive properties to simplify algebraic expressions.

A.4(C)

• The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. The student is expected to connect equation notation with function notation, such as \( y = x + 1 \) and \( f(x) = x + 1 \).

Objective 6

8.6(A)

• The student uses transformational geometry to develop spatial sense. The student is expected to generate similar figures using dilations including enlargements and reductions.

8.6(B)

• The student uses transformational geometry to develop spatial sense. The student is expected to graph dilations, reflections, and translations on a coordinate plane.

8.7(D)

• The student uses geometry to model and describe the physical world. The student is expected to locate and name points on a coordinate plane using ordered pairs of rational numbers.

Objective 8

8.8(A)
• The student uses procedures to determine measures of three-dimensional figures. The student is expected to find lateral and total surface area of prisms, pyramids, and cylinders using concrete models and nets (two-dimensional models).

8.8(B)

• The student uses procedures to determine measures of three-dimensional figures. The student is expected to connect models of prisms, cylinders, pyramids, spheres, and cones to formulas for volume of these objects.

8.8(C)

• The student uses procedures to determine measures of three-dimensional figures. The student is expected to estimate measurements and use formulas to solve application problems involving lateral and total surface area and volume.

8.9(A)

• The student uses indirect measurement to solve problems. The student is expected to use the Pythagorean Theorem to solve real-life problems.

8.9(B)

• The student uses indirect measurement to solve problems. The student is expected to use proportional relationships in similar two-dimensional figures or similar three-dimensional figures to find missing measurements.

8.10(A)

• The student describes how changes in dimensions affect linear, area, and volume measures. The student is expected to describe the resulting effects on perimeter and area when the dimensions of a shape are changed proportionally.

8.10(B)

• The student describes how changes in dimensions affect linear, area, and volume measures. The student is expected to describe the resulting effect on volume when dimensions of a solid are changed proportionally.
Important Vocabulary Words

Adapted from Bennett (2007) and Burger (2007).

Objective 2

Algebraic expression – an expression that contains at least one variable

Associative Property of Addition – the property that states that for all real numbers a, b, and c, the sum is always the same, regardless of their grouping

Associative Property of Multiplication – the property that states that for all real numbers a, b, and c, the product is always the same, regardless of their grouping

Commutative Property of Addition – the property that states that two or more numbers can be added in any order without changing the sum

Commutative Property of Multiplication – the property that states that two or more numbers can be multiplied in any order without changing the product

Continuous graph – a graph made up of connected lines or curves

Coordinate – one of the numbers of an ordered pair that locate a point on a coordinate graph

Coordinate plane – a plane formed by the intersection of a horizontal number line called the x-axis and a vertical number line called the y-axis

Correlation – a measure of the strength and direction of the relationship between two variables or data sets

Cubic polynomial – a polynomial of degree 3

Dependent variable – the output of a function; a variable whose value depends on the value of the input, or independent variable

Discrete graph – a graph made up of unconnected points

Distributive Property – the property that states if you multiply sum by a number you will get the same result if you multiply each addend by that number and then add the products.

Domain – the set of all possible input values of a function
Equation notation – the notation used to state that two expressions are equivalent

Factor – a number or expression that is multiplied by another number or expression to get a product

Factoring – the process of writing a number or algebraic expression as a product

Function – an input – output relationship that has exactly one output for each input

Function notation – the notation used to describe a function

Independent variable – the input of a function; a variable whose value determines the value of the output, or dependent variable

Integer – the set of whole numbers and their opposites

Irrational number – a number that cannot be expressed as a ratio of two integers or as a repeating or terminating decimal

Like terms – terms with the same variables raised to the same exponents

Linear function – a function whose graph is a straight line

Order of operations – a process for evaluating expressions: parenthesis, exponents, multiplication and division (from left to right), then addition and subtraction (from left to right)

Parabola – the shape of the graph of a quadratic function

Parent function – the simplest function with the defining characteristics of the family

Quadratic function – a function of the form \( y = ax^2 + bx + c \), where \( a \neq 0 \)

Range – the difference between the greatest and least values in a data set

Rational number – any number that can be expressed as a ratio of two integers

Real number – a rational or irrational number

Relation – a set of ordered pairs

Scatterplot – a graph with points plotted to show a possible relationship between two sets of data

Slope – a measure of the steepness of a line
Slope-intercept form – a linear equation written in the form $y = mx + b$ where $m$ represents slope and $b$ represents the $y$-intercept

Vertical line test – a test used to determine whether a relation is a function. If any vertical line crosses the graph of a relation more than once, the relation is not a function

X-axis – the horizontal axis in a coordinate plane

Y-axis – the vertical axis in a coordinate plane

Y-intercept – the $y$-coordinate of the point where a graph intersects the $y$-axis

**Objective 6**

Center of dilation – the point of intersection of lines through each pair of corresponding vertices in a dilation

Center of rotation – the about which a figure is rotated

Dilation – a transformation that enlarges or reduces a figure

Image – a shape that results from a transformation of a figure known as the preimage

Preimage – the original figure in a transformation

Primes – symbols used to label the image in a transformation

Proportion – an equation that states that two ratios are equivalent

Quadrant – the $x$- and $y$-axes divide the coordinate plane into four regions, each called a quadrant

Reflection – a transformation of a figure that flips the figure across a line

Rotation – a transformation in which a figure is turned around a point

Scale factor – the ratio used to enlarge or reduce similar figures

Similar figures – figures with the same shape but not necessarily the same size

Transformation – a change in the size or position of a figure

Translation – a movement (slide) of a figure along a straight line

**Objective 8**
Cone – a three-dimensional figure with one vertex and one circular base

Cylinder – a three-dimensional figure with two parallel, congruent circular bases connected by a curved lateral surface

Indirect measurement – the technique of using similar figures and proportions to find a measure

Hypotenuse – in a right triangle, the side opposite the right angle

Lateral area – the sum of the areas of the lateral faces of a prism or pyramid, or the area of the lateral surface of a cylinder or cone

Lateral faces – a face of a prism or pyramid that is not a base

Lateral surface – the curved surface of a cylinder or cone

Leg – in a right triangle, the sides that include the right angle

Net – an arrangement of two-dimensional figures that can be folded to form a polyhedron

Prism – a polyhedron that has two congruent, polygon-shaped bases and other faces that are all parallelograms

Pyramid – a polyhedron with a polygon base and triangular sides that all meet at a common vertex

Pythagorean Theorem – in a right triangle, the square of the length of the hypotenuse is equal to the sum of the lengths of the squares of the legs

Sphere – a three-dimensional figure with all points the same distance from the center

Total surface area – the sum of the areas of all the faces and curved surfaces of a three-dimensional figure

Volume – the number of cubic units needed to fill a given space
APPENDIX F

Pre- and Post-Assessments
Pre-Test
Objectives 2, 6, 8

1. If a circle has center (4, 2) and a radius of 3, in which quadrant(s) does the circle lie?

2. The math club is raising money selling t-shirts. The function \( f(s) = 15s \) describes the amount of money, in dollars, that the math club will make for selling \( s \) shirts. What is the domain and range of this function?

   Domain: _______________________
   Range: _______________________

3. A cylindrical can has a radius of 1.5 inches and a height of 6 inches. What is the total surface area and volume of the can?

   \( SA = \) _______________________
   \( V = \) _______________________

4. The graph shows the height, in feet, of a football \( t \) seconds after it was thrown. Based on the graph, what is the minimum height of the ball?

   _______________________

   What was the maximum height of the ball? _______________________

   How long was the ball in the air?
   _______________________

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5. Triangles ABC and A'B'C' are shown on the coordinate plane. Which is the best description of the transformation used to arrive at A'B'C' from ABC?

6. What is the effect on the area of a circle if the radius is doubled?

7. If \( f(x) = 2x^2 + 3x - 5 \), what is \( f(-1) \)?

8. Triangle ABC is shown in the coordinate plane. If ABC is translated so that B is mapped to B', what is the coordinate of:

   \( A' \)?

   \( C' \)?

9. Jenn's house is 250 feet north and 600 feet east of the gas station. About how far is it traveling in a straight line from Jenn's house to the gas station?
10. A rectangle is formed with a length of $x + 2y$ and a width of $2x + y$.

What is the perimeter of this rectangle? ______________________

What is the area? _________________________________
Pre-Test Key
Objectives 2, 6, 8

1. If a circle has center (4, 2) and a radius of 3, in which quadrant(s) does the circle lie?
   \[\text{I and IV}\]

2. The math club is raising money selling t-shirts. The function \( f(s) = 15s \) describes the amount of money, in dollars, that the math club will make for selling \( s \) shirts. What is the domain and range of this function?
   
   Domain: \( \{1, 2, 3, 4, \ldots\} \)
   
   Range: \( \{15, 30, 45, \ldots\} \)

3. A cylindrical can has a radius of 1.5 inches and a height of 6 inches. What is the total surface area and volume of the can?
   
   \( \text{SA} = 70.7 \text{ in}^2 \)
   
   \( V = 42.4 \text{ in}^3 \)

4. The graph shows the height, in feet, of a football \( t \) seconds after it was thrown. Based on the graph, what is the minimum height of the ball? \( 0 \text{ feet} \)

   What was the maximum height of the ball? \( 8 \text{ feet} \)

   How long was the ball in the air?
   \( 8 \text{ seconds} \)
5. Triangles ABC and A'B'C' are shown on the coordinate plane. Which is the best description of the transformation used to arrive at A'B'C' from ABC?

*Reflection across the y-axis*

6. What is the effect on the area of a circle if the radius is doubled? *The new area is 4 times as large*

7. If \( f(x) = 2x^2 + 3x - 5 \), what is \( f(-1) \)? *-6*

8. Triangle ABC is shown in the coordinate plane. If ABC is translated so that B is mapped to B', what is the coordinate of:

   A'? *(6, 1)*

   C'? *(7, 4)*

9. Jenn's house is 250 feet north and 600 feet east of the gas station. About how far is it traveling in a straight line from Jenn's house to the gas station? *650 feet*

10. A rectangle is formed with a length of \( x + 2y \) and a width of \( 2x + y \).

   What is the perimeter of this rectangle? *6x + 6y*

   What is the area? *\( 2x^2 + 5xy + 2y^2 \)*
Post-Test
Objectives 2, 6, 8

1. If a circle has center (3, 1) and a radius of 2, in which quadrant(s) does the circle lie?
   ____________________________________________

2. The math club is raising money selling t-shirts. The function f(s) = 10s describes the amount of money, in dollars, that the math club will make for selling s shirts. What is the domain and range of this function?
   Domain: ____________________________
   Range: ____________________________

3. A cylindrical can has a radius of 1.25 inches and a height of 5 inches. What is the total surface area and volume of the can?
   SA = ____________________________
   V = ____________________________

4. The graph shows the height, in feet, of a football t seconds after it was thrown. Based on the graph, what is the minimum height of the ball?
   ____________________________

   What was the maximum height of the ball?
   ____________________________

   How long was the ball in the air?
   ____________________________
5. Triangles ABC and A'B'C' are shown on the coordinate plane. Which is the best description of the transformation used to arrive at A'B'C' from ABC?

6. What is the effect on the area of a circle if the radius is tripled?

7. If \( f(x) = 4x^2 + 10x - 6 \), what is \( f(-5) \)?

8. Triangle ABC is shown in the coordinate plane. If ABC is translated so that B is mapped to B', what is the coordinate of:

   \[ A' \]?

   \[ C' \]?

9. Jenn's house is 300 feet north and 800 feet east of the gas station. About how far is it traveling in a straight line from Jenn's house to the gas station?
10. A rectangle is formed with a length of $3x + y$ and a width of $x + y$.

What is the perimeter of this rectangle? ________________________

What is the area? ________________________________
Post-Test Key
Objectives 2, 6, 8

1. If a circle has center (3, 1) and a radius of 2, in which quadrant(s) does the circle lie?

* I and IV *

2. The math club is raising money selling t-shirts. The function \( f(s) = 10s \) describes the amount of money, in dollars, that the math club will make for selling \( s \) shirts. What is the domain and range of this function?

   Domain: \( \{1, 2, 3, 4, \ldots \} \)

   Range: \( \{10, 20, 30, 40, \ldots \} \)

3. A cylindrical can has a radius of 1.25 inches and a height of 5 inches. What is the total surface area and volume of the can?

   \[
   SA = 49.1 \text{ in}^2 \\
   V = 24.5 \text{ in}^3
   \]

4. The graph shows the height, in feet, of a football \( t \) seconds after it was thrown. Based on the graph, what is the minimum height of the ball? 0 feet

   What was the maximum height of the ball? 7 feet

   How long was the ball in the air? 6 seconds
5. Triangles ABC and A'B'C' are shown on the coordinate plane. Which is the best description of the transformation used to arrive at A'B'C' from ABC?

Reflection across the x-axis

6. What is the effect on the area of a circle if the radius is tripled? The new area will be 9 times as large.

7. If \( f(x) = 4x^2 + 10x - 6 \), what is \( f(-5) \)? 44

8. Triangle ABC is shown in the coordinate plane. If ABC is translated so that B is mapped to B', what is the coordinate of:

\[ A' \approx (-5, 1) \]

\[ C' \approx (-4, 4) \]

9. Jenn’s house is 300 feet north and 800 feet east of the gas station. About how far is it traveling in a straight line from Jenn’s house to the gas station? 854.4 feet

10. A rectangle is formed with a length of \( 3x + y \) and a width of \( x + y \).

What is the perimeter of this rectangle? \( 8x + 4y \)

What is the area? \( 3x^2 + 4xy + y^2 \)
APPENDICES REFERENCES


