Algebraic Habits of Mind
Student Needs and Solutions

A PROPOSAL for a PROJECT in MATHEMATICS

by

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Algebra 1 is the first course high school students study where problem solving changes from concrete to abstract. Students in Algebra 1 need to have persistence to discover these "new" relationships and to explore mathematical strategies, properties and relationships. Sadly, many students are not well equipped to enter the Algebra 1 classroom. Their problem solving skills, both affective and academic, are weak. Many students have had little success with mathematical exploration, and are content to be instructed, rather than engaging actively with their own learning. In fact the failure rate of Algebra 1 students is the highest of all high school courses in virtually all suburban high schools in the author's region.

This author will research the habits of mind most necessary for success in the Algebra 1 curriculum and will design a preparatory Algebra course that will be available to the author's school district. This course will be offered in the summer prior to students' enrollment in Algebra 1 in the school year.

It is expected that students will be successful in their Algebra 1 course the school year following their experience with the curriculum emphasizing algebraic habits of mind. The students' grades will measure their algebraic strengths as well as their reflections demonstrating the mathematical confidence gained interacting with the curriculum.
Algebraic Habits of Mind

Student Needs and Solutions

Problem solving . . . can serve as a vehicle for learning new mathematical ideas and skills. . . . A problem-centered approach to teaching mathematics uses interesting and well-selected problems to launch mathematical lessons and engage students. In this way, new ideas, techniques, and mathematical relationships emerge and become the focus of discussion. Good problems can inspire the explorations of important mathematical ideas, nurture persistence, and reinforce the need to understand and use various strategies, mathematical properties, and relationships.

--Principles and Standards for School Mathematics (NCTM, 2003, p. ix)

Algebra 1 has the highest failure rate of all classes at a local large suburban high school. This statistic bears out in virtually all high schools in Texas and across the nation. In fact, according to Los Angeles Schools' Superintendent Roy Romer, "[Algebra 1] triggers dropouts more than any single subject" (Los Angeles Times, 2006). Further, according to Los Angeles Times' Duke Helfand "... thousands . . . across the district also failed algebra" (Daily Howler, 2006). Students tell of their difficulty with Algebra 1, that they do not understand the mathematics and that it is one of the most important courses that a high school student will study. Algebra 1 builds the foundation for all future mathematical and scientific studies. Students' success or failure in Algebra I will affect their entire scholastic future.

Students who have strong basic mathematical habits of mind are able to: solve problems, communicate effectively, reason logically and make connections between mathematics and other disciplines (UGACOE, 2007). Students who attain these skills will be successful in Algebra 1 and future mathematical endeavors. This project will
produce a curriculum for preparatory Algebra with a concentration on developing algebraic habits of mind necessary to succeed in the high school curriculum.

LITERATURE REVIEW

Habits of mind have been studied by many researchers (i.e. Marzano, 1998; Borowski, Carr, Rellinger and Pressley, 1990; Paris and Winograd, 1990). Habits of mind can be described as those well-established thinking behaviors and patterns characteristic of methodical and logical thinking. Specifically, these behaviors and patterns are identified as problem solving, communicating, reasoning, and making connections (UGACOE, 2007).

Researchers report that a strong foundation in algebraic habits of mind provides a strong framework that can be built upon in all future mathematical studies. Driscoll (1999) lists the habits of mind necessary for success in algebraic problem solving:

- Doing-undoing
- Building rules to represent functions
- Abstracting from computation (p. 15)

Burke (2002) lists the fundamental components of algebraic thinking as:

- Understanding patterns, relations, and functions
- Representing and analyzing mathematical situations and structures using algebraic symbols
- Using mathematical models to represent and understand quantitative relationships
- Analyzing change in various contexts (pp. 2-5)

Levasseur and Cuoco (2003) list the most significant and useful mathematical habits of mind as:

- Guessing is not necessarily bad-we all do it!
- Challenge solutions, even correct ones
- Look for patterns
- Conserve memory
- Specialize-Sometimes everything is special
• Use alternative representations
• Carefully classify
• Think algebraically (pp. 28-33)

This project will investigate these and other authors' interpretations of the most useful and teachable habits of mind. Habits of mind most necessary for a beginning Algebra 1 student will be explored. This project will produce a curriculum that addresses these habits of mind with a focus on Algebra 1. Topics will be chosen to encourage use of varied habits of mind. Topics that encourage habits that can be well learned comma understood and transitioned well to algebra, yet studied within a context of a curriculum of 2-3 weeks will be especially included. The topics will be chosen and then organized in a manner that builds on previous learned skills and then challenges the students to apply their knowledge skills in novel and advanced ways.

Presented as a summer Algebra 1 mathematics preparatory course, this project will be an exciting and innovative program for a school district to offer. This curriculum will build a foundation for learning Algebra 1 while also building student confidence and persistence. The goal of this project is to produce a problem based curriculum that focuses on building algebraic habits of mind. A possible effect of this curriculum is greater success of students in Algebra 1 and future mathematical endeavors. Future research will test this effect.

Some of the topics of study that have inspired this author are as follows:

• The Locker Problem (Driscoll, 1999, pp. 10-11) and Lucky Lockers (Driscoll, 1999, pp. 62-63)
• Postage Stamp Problem (Driscoll, 1999, pp. 24-26)
• A Crawling Snail (Driscoll, 1999, p. 31)
• Lots of Squares (Driscoll, 1999, pp. 39-41)
• Clocking (Driscoll, 1999, p. 57)
• Towering Numbers (Driscoll, 1999, pp. 100-102)
• Would You Work for Me? (Burke, 2002, pp. 26, 63-65)
• Crossing the River (Herbert, 1999, pp. 124-127)

Naturally, each topic will be explored in ways beyond the explanation by the given authors. In the context of the preparatory Algebra curriculum, class discussions will be encouraged. Multiple representations, use of technology, and reflections in journals concerning conjectures, solutions, and mistakes will be encouraged.

The above listed problem scenarios have been chosen for one or more of the following reasons listed:

• Problem can be modeled concretely, visually, and abstractly.
• Multiple representations may be used, i.e. verbal, graph, function, spreadsheet, etc.
• Technology is useful in the interpretation, modeling, and solving of the problem.
• Basic structure of problem is understood by many, but the interpretation, modeling, and solving of the problem may be different for different students.
• Problem lends itself to extension and connection to other problems.
• Students may formulate conjectures and guesses and then test.

PLANNED ACTIONS

This project will begin with researching algebraic habits of mind. Problems to be included in the curriculum will be carefully chosen to reflect those habits of mind most necessary for student success in Algebra. Also, situations will be chosen that foster problem solving, communication, reasoning, and making connections in useful, interesting and novel ways.

Lessons from this project will be presented at the annual \((\text{ME})^2\) by the Sea Conference to teachers and future teachers. Participants will be asked to evaluate the effectiveness of the problem scenario(s) based on their teaching experience. Participants will be asked for feedback in two areas: first, did the lesson provide for application of the habit(s) of mind intended, and secondly, did the lesson provide for
deeper understanding of mathematics in a novel and interesting manner. Feedback from teachers will be used to adjust the curriculum.

After this project is completed the author plans many applications. This curriculum will first be presented to the author's school district for possible implementation in the late summer of 2007. It is the author’s intention that the course would be offered to students who will be enrolled in Algebra 1 during fall 2007. These students may self-identify, or may be referred by their counselors. The course will be offered in early August, before the start of the academic school year. Students will be enrolled in the preparatory Algebra 1 course for three hours per day, for two weeks. This head start on the school year will give students an edge, both mathematically by building mathematic skills, and affectively by building confidence in mathematics.

In the future, the author intends to offer this course privately. This course will be offered to students in the surrounding region that self identify. The course would still be offered as three hours per day for two weeks, but the author plans to build the program to offer variations of the curriculum. Variations may include, but are not limited to, different levels of mathematics courses to be offered to students who:

- Have strong mathematics skills but are not content with their level of mathematics understanding,
- Are generally weak in mathematics and need a curriculum deep in fundamental mathematical understanding, or
- Are preparing for state or national testing.
TIME LINE

January 16- March 30, 2007  Prepare proposal
March 26- March 30, 2007  Distribute proposal to committee members
April 2-6, 2007  Project proposal defense
April 2-May 31, 2007  Research habits of mind and curriculum based
June 1-June 15, 2007  on habits of mind
June 15-20, 2007  Write curriculum using feedback from (ME)$^2$ by
July 2-6, 2007  the Sea Conference
August 4, 2007  Distribute project to committee members
                   Project defense
                   Graduation

In summation, the research for this project will focus on the habits of mind necessary
to encourage mathematical competency in Algebra 1. The project will be a 2-3 week
curriculum that will foster the algebraic habits of mind found necessary for Algebra 1
understanding. It is the intention of this author to present this curriculum to the district of
residence in the hopes that students may acquire these needed skills. This curriculum
will challenge students and prepare them for a meaningful experience with Algebra 1.
References


Los Angeles Times. (2006). A formula for failure in L.A. schools: Because they can't pass algebra, thousands of students are denied diplomas. Many try again and