Deliberate Practice for Algebra 1 with the TI-Nspire® Calculator: Electronic Supplemental Resources

A PROPOSAL for a PROJECT in MATHEMATICS

by

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Style: APA
Abstract

The proposed project will develop TI-Nspire programs that support deliberate practice of the procedures associated with one- and two-variable linear equations. TI-Nspire calculator was selected as the platform for the deliberate practice because it is handheld, it contains features that support differentiated practice, and it is widely available to teachers in Corpus Christi, TX. Following recommendations by Brabeck and Jeffrey (2011), the project aims to design programs for the handheld TI-Nspire® device that provide (1) practice at each individual student’s level, (2) timely and descriptive feedback, and (3) repeated opportunities to practice. Cognitive Load Theory also informs the design of the deliberate practice, with the goal of supplementing existing Algebra I curricula to support automation of students’ supporting schema. Current Algebra 1 curricula and existing electronic resources are reviewed, and all would be supplemented well by automated deliberate practice.
Introduction

Nothing is fun until you’re good at it. . . . Tenacious practice, practice, practice is crucial for excellence; rote repetition is underrated in America. Once a child starts to excel at something – whether it’s math, piano, pitching, or ballet – he or she gets praise, admiration, and satisfaction. This builds confidence and makes the once not-fun activity fun. This in turn makes it easier for the parent to get the child to work even more (Chua, 2011, p. 29).

Deliberate practice consists of activities purposely designed to improve performance. These activities typically require effort and are not enjoyable (Gobet & Campitelli, 2007, p. 160).

As an Algebra I teacher in Corpus Christi, TX, the author has often seen the negative effects of students’ lack of practice in foundational mathematics skills. Anecdotally, many Algebra 1 teachers observe that many students lack a solid skills base for numeric operations and algebraic processes. This contention is supported by middle school mathematics education research, with Foegen and Deno (2001), for example, finding that measures of basic number facts account for 44% of variation in state standardized test performance among middle school mathematics students. Standardized test data also indicates that a substantial portion of Algebra 1 students lack fluency with linear functions, equations, and inequalities. At the state level, Texas students scored an average of 64% and 65% on the Texas Assessment of Knowledge and Skills Objectives 3 and 4, which assess Linear Functions and Linear Equations and Inequalities, respectively (Texas Education Agency (TEA), 2010d). Corpus Christi Independent School District (CCISD) fared more poorly, scoring 59% and 61%, respectively (TEA, 2010c).
A lack of proficiency with basic procedures can slow students' progress towards mastering objectives that incorporate application of those skills. In particular, Algebra 1 instruction would benefit from deliberate practice that reinforces basic Algebra 1 skills such as solving and manipulating equations.

Broadly, the purpose of this project is to design and create programs that generate unlimited deliberate practice to supplement existing Algebra 1 curriculum. The content focus of the practice will consist of one- and two-variable linear equations and systems of two-variable linear equations.

The intended outcome of the project is to produce electronic materials for the TI-Nspire calculator that will help Algebra 1 instructors to automate students' schemas associated with linear equations and linear functions, reducing the students' cognitive load when reasoning with these topics. This rests on the conviction that, if successfully implemented by teachers, manipulation and classification of linear equations, both in one and two variables, should be reduced to a relatively low intrinsic load for students, and this would provide opportunities to facilitate the deeper understandings of these topics. The guiding principles of the project are as follows:

1. Linear equations and linear functions are an important topic to be addressed in an Algebra 1 curriculum.
2. The use of deliberate practice to automate schema in Algebra 1 is pedagogically sound.
3. Current curricula would be supplemented well by more deliberate practice.
4. The TI-Nspire provides a novel and effective platform to leverage modern technology to implement deliberate practice.
Related Works and Justification

**Content Focus**

The content focus of this project was driven by two major resources: (1) state and national standards which are meant to guide instruction through establishing common learning objectives, and (2) relevance of linear equations and functions to algebraic thinking.

**Alignment to State and National Standards**

This project directly addresses both Supporting Standards and Readiness Standards as set forth by the TEA. Readiness Standards are meant to be the focus of a given course, and they will constitute 60-65% of assessment focus (TEA, 2010b). Supporting Standards refer to content that is considered prior knowledge and/or content that will be stressed in more depth in a future math class. Given their broad categorization, Supporting Standards are roughly twice as numerous as Readiness Standards. Table 1 summarizes the Algebra 1 Texas Essential Knowledge and Skills (TEKS) which are most relevant to the content focus of the project (TEA, 2010a).

To clarify some vocabulary: the listed TEKS refer to linear equations in one variable simply as linear equations, and they refer to linear equations in two variables as linear functions. Interestingly enough, the standards refer to a system of two-variable linear equations as a system of linear equations rather than a system of linear functions. Unless otherwise stated, linear equations, linear functions, and linear systems will refer to one-variable linear equations,
two-variable linear equations, and systems of two-variable linear equations, respectively.

Table 1

<table>
<thead>
<tr>
<th>TEKS</th>
<th>Equations</th>
<th>Functions</th>
<th>Systems</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.2.A Identify and sketch the general forms of linear ( y = x ) and quadratic ( y = x^2 ) parent functions.</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>*A.4.A Find specific function values, simplify polynomial expressions, transform and solve equations, and factor as necessary in problem situations.</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>A.4.B Use the commutative, associative, and distributive properties to simplify algebraic expressions.</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>*A.5.C Use, translate, and make connections among algebraic, tabular, graphical, or verbal descriptions of linear functions.</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>A.6A Develop the concept of slope as rate of change and determine slopes from graphs, tables, and algebraic representations.</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>A.6.D Graph and write equations of lines given characteristics such as two points, a point and a slope, or a slope and ( y )-intercept.</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>A.6.E Determine the intercepts of the graphs of linear functions and zeros of linear functions from graphs, tables, and algebraic representation.</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>*A.7.B Investigate methods for solving linear equations and inequalities using [concrete] models, graphs, and the properties of equality, select a method, and solve the equations and inequalities.</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>A.7.C Interpret and determine the reasonableness of solutions to linear equations and inequalities.</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>*A.8.B Solve systems of linear equations using [concrete] models, graphs, tables, and algebraic methods.</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>A.8.C Interpret and determine the reasonableness of solutions to systems of linear equations.</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>

*Note. Yes/No indicates whether or not the particular standard applies to linear equations, functions, or systems. * indicates a Readiness Standard.*
In addition to state standards, many teachers, researchers, and organizations have pointed to linear equations, functions, and systems as fundamental Algebra 1 topics. In particular, the National Council of Teachers of Mathematics (NCTM) has established standards for Algebra in grades 9-12.

Table 2 summarizes NCTM standards that apply to linear equations, functions, and systems (Burke, Erickson, Lott, & Obert, 2001).

Table 2

<table>
<thead>
<tr>
<th>NCTM Standards</th>
<th>Equations</th>
<th>Functions</th>
<th>Systems</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interpret representations of functions of two variables.</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency-mentally or with paper and pencil in simple cases and using technology in all cases.</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Use symbolic algebra to represent and explain mathematical relationships.</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Understand relations and functions and select, convert flexibly among, and use various representations for (functions).</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

*Note. Yes/No indicates whether or not the particular standard applies to linear equations, functions, or systems.*

The Common Core State Standards Initiative (CCSSI) also established a set of Algebra 1 standards that are meant to provide a “clear and consistent framework” (Common Core State Standards Initiative, 2010) that informs Algebra 1 instruction. The relevant Standards are summarized in Table 3. While both the NCTM and the CCLS standards reflect an overall emphasis on problem solving, both sets of standards draw on schemas that can be automated by deliberate practice. The expectation is that students who develop a facility with manipulating
linear equation in one and two variables will be better able to problem solve within these domains.

Table 3

Summary of Relevant Common Core State Standards Initiative (CCSSI) Algebra Standards (CCSSI, 2010)

<table>
<thead>
<tr>
<th>CCSSI Algebra Standards</th>
<th>Equations</th>
<th>Functions</th>
<th>Systems</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-SSE.1. Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients. Interpret complicated expressions by viewing one or more of their parts as a single entity.</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>A-CED.1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>A-CED.2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>A-CED.4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>A-REI.3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>A-REI.6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>A-REI.10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>F-IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
</tbody>
</table>

Note. Yes/No indicates whether or not the particular standard applies to linear equations, functions, or systems.
Relevance of One- and Two-Variable Linear Equations.

The book *Fostering Algebraic Thinking* (Driscoll, 1999) discusses many fundamental Algebra 1 topics, and these are supported by emphasis on linear equations, functions, and systems. Driscoll focuses on three fundamental algebraic habits of mind, one of which is “doing-undoing” (p. 1). According to Driscoll, “effective algebraic thinking sometimes involves reversibility…the capacity not only to use a process to get to a goal, but also to understand the process well enough to work backward from the answer to the starting point” (pp. 1-2). Linear equations, functions, and systems provide a broad context for examining the doing-undoing habit of mind. Students can explore doing-undoing by transforming linear functions, such as converting between the standard, point-slope, and slope-intercept forms of a linear function. As a different example, the algorithm for solving linear equations provides an excellent context for discussing inverse operations and their relation to doing-undoing.

More generally, Driscoll (1999) stresses the importance of helping students develop symbol sense, and it is important to note that linear equations, functions, and systems are all germane to symbol sense. Because “symbolic representation and manipulation are the lifeblood of algebra” (p. vii), Driscoll devotes substantial portions of the text to exploring how teachers can facilitate students’ development of symbol sense, highlighting several “facets of symbol sense” (p. 120). Among those facets, the text expresses a need for students to be “able to inspect an algebraic operation and predict the form of the result” (p. 121). This ability is exercised when students are required to distinguish between
desired results in the contexts of one- and two-variable linear equations. For example, given a problem involving a one-variable linear equation, the desired result is almost always a single number, the correct value of the variable. In contrast, problems involving two-variable linear equations have a wider variety of desired results. Given a two-variable linear equation, students could be required to produce a graph, a table, an alternate form, or they could even be required to find specific quantities of interest such as the y-intercept or slope. Importantly, two- and one-variable linear equations even have solutions that have different forms: ordered pairs rather than a single number. Practicing with manipulating one- and two-variable equations gives students a variety of opportunities to connect a problem to its desired result state.

Driscoll (1999) also stresses the importance of multiple representations, and linear functions form a solid basis for exploring connections between the various representations of a function. According to Driscoll, “a fluency linking and translating among multiple representations seems to be critical in the development of algebraic thinking” (p. 141) and “one characteristic of a successful solver of algebra word problems is the ability to translate from verbal, tabular, graphical, and diagrammatic representation into symbolic representations that can be manipulated” (p. 145). Two-variable linear equations lend themselves to multiple representations, so they provide the perfect context for students to translate between those various representations.
Pedagogical Approach

The approach of the project is to create deliberate practice for students with the hope of creating automaticity of the associated procedural skills. The conceptual framework of Cognitive Load Theory (CLT) guides this strategy for general knowledge acquisition, and research indicates more specifically that deliberate practice improves academic performance (Brabeck & Jeffrey, 2011).

Cognitive Load Theory

A basic premise of Cognitive Load Theory (CLT) is that a student’s cognitive resources are limited. Learning requires active use of working memory, which can process only a small number of “novel interacting elements” (Paas, Renkyl, & Sweller, 2003). For teachers, limited cognitive resources imply that learning can be impeded when students’ working memory is taxed with too many new topics to process simultaneously. This impedance is commonly referred to as cognitive overload.

Complex cognitive tasks can be accomplished when long term memory supplements working memory. According to Paas, Renkyl, and Sweller (2003):

*Long-term memory* provides humans with the ability to vastly expand [working memory’s] processing ability. This memory store can contain vast numbers of *schemas*—cognitive constructs that incorporate multiple elements of information into a single element with a specific function. Schemas can be brought from long-term to working memory. . . . The automation of those schemas so that they can be processed unconsciously further reduces the load on working memory. It is by this process that human cognitive architecture handles complex material that appears to exceed the capacity of working memory. (p. 3)

In other words, complex cognitive tasks can be more manageable if the associated schemas are automated. In addition to recognizing students’ limited
cognitive resources, CLT separates cognitive loads into three categories: intrinsic, germane, and extraneous. Paas, Renkl, and Sweller (2004) elaborate further:

CLT distinguishes between three types of cognitive load: intrinsic, extraneous, and germane. The load is called 'intrinsic' if it is imposed by the number of information elements and their interactivity. If it is imposed by the manner in which the information is presented to learners and by the learning activities required of learners, it is called 'extraneous' or 'germane'. (p. 2)

The germane load of an instructional task includes the students’ ability to connect related topics. For example, take a set of related topics such as the algorithm for solving a linear equation, the nature of inverse operations, and the rules for operations with integers. The latter two topics arise in the application of the first. In this case, these latter topics represent intrinsic cognitive loads that could be considered independently, and the germane cognitive load could consist of the application of these topics to solve a linear equation.

With regard to equation solving, if students exhibit difficulty applying schemas associated with integral operations and inverse operations, then those topics become cognitively taxing and will reduce the resources that the student can dedicate to germane load. Solving relatively simple two-step equations might be taxing yet realistic, but solving more complex equations would require additional schemas and likely involve cognitive overload. Typically, teachers would like students to be able to focus on the germane load of a lesson, which often incorporates higher order thinking; the overall cognitive load reduces greatly when the skills involved with the intrinsic load are automated. In this way,
focus on germane load when learning mathematics can be supported by developing automaticity of relevant skill sets.

**Deliberate Practice**

Deliberate practice is a strategy for effectively lowering the total cognitive load of tasks, because after practice, relevant “elements will become subsumed into a schema, which can be treated as a single element in working memory” (Paas & van Gog, 2006, p. 87). However, teachers must take care to distinguish deliberate practice from rote repetition. According to the American Psychological Association’s General Recommendations to Teachers Regarding Practice (Brabeck & Jeffrey, 2011), there are several features of effective deliberate practice:

a. Teachers must point out when students’ practice has actually improved their performance.

b. Teachers should ensure that practice is within the student’s ability level.

c. Teachers should provide students with timely and descriptive feedback.

d. “Students should have repeated opportunities to practice a task through practicing other tasks like it.” (p. 3)

These recommendations form a pedagogical basis for designing deliberate practice, and they can be applied well to this project’s program design. Results of a user’s performance can be summarized at the end of a practice session, and students will be able to quantify progress in levels and percentages. The program can then adjust the difficulty of problems automatically depending on users’ performance, and provide immediate feedback on user input. Second chances, instructional responses to common errors, and correct responses make
the feedback very descriptive, and of course the general topics of linear
equations, functions, and systems of linear equations provide students with
ample opportunities to practice similar tasks repetitively.

With regard to practicing “a task through practicing other tasks like it”
(Brabeck & Jeffrey, 2011, p. 3), manipulating a linear equation, whether in one
variable or two, typically involves applying inverse operations along with the
properties of equality in the context of numerical coefficients. So whether a
student is manipulating a linear equation in one or two variables, the two tasks
resemble each other in fundamental ways. In terms of the procedural steps,
rearranging a two-variable equation to isolate a variable is very much like solving
a one-variable equation. In a similar way, systems of linear equations provide an
excellent context for rearranging equations and also for solving one-variable
equations. The loosely hierarchical nature of the three topics lends itself well to
deliberate practice.

Supplemental Value

The purpose of this section is to provide evidence that the current Algebra
1 curriculum in Corpus Christi Independent School District could benefit from an
additional resource that provides students with deliberate practice. The project
will likely have similar supplemental value for Algebra 1 teachers in other locales.
The following sub-sections will outline the program’s supplemental value to
existing Algebra 1 curricula and to existing electronic resources as well.
Review of a Current Curriculum

It is an educational truism that no single curriculum is all-encompassing and that teachers may need to supplement any curriculum with additional resources. This project is intended to be a resource to supplement existing curricula, so as an example of a curriculum that could be supplemented with this program, the author examined the Holt Algebra 1 curriculum (Burger, et al., 2007), as adopted by CCISD in 2007. The author chose this particular resource for several reasons: (1) it is available to every Algebra 1 teacher in CCISD; (2) it is listed as a primary resource in CCISD’s scope and sequence documents for Algebra 1; (3) it addresses all of the Algebra 1 TEKS; (4) the author has extensively used the text and found it to be useful as a primary instructional resource. For illustration purposes, consider Holt 2-3: Solving Two-Step and Multi-Step Equations (Burger, et al., 2007, p. 96). This book section follows two others that cover one-step linear equations that involve reaching a solution through a single operation that is always addition, subtraction, multiplication, or division. The section also comes just before the section in which students are required to solve one variable equations having variables on both sides. For the purposes of this discussion, a two-step equation shall always refer to a one-variable linear equation in that requires one operation of addition/subtraction and one operation of multiplication/division to generate a solution (e.g., \(2x - 3 = 5\)).

Solving two step equations involves a small number of concepts that are fundamental in Algebra 1: properties of equality (germane load), inverse operations (germane load), and in some cases integer operations (intrinsic load). Teachers can avoid extraneous cognitive loads such as fractions, decimals, and
commutation by carefully selecting equations with integral coefficients and solutions. By deliberately choosing characteristics of practice problems, the teacher can facilitate students’ emphasis on the germane cognitive loads and limit the intrinsic load to one or two topics.

*Need for more practice problems.*

In this single book section and the accompanying worksheets, there are 29 two-step equations, 15 of which have integer solutions and coefficients and explicitly fit the pattern “$ax \pm b = c$.” The other 14 two-step equations fit a different pattern; therefore, these equations would introduce the additional intrinsic loads of the commutative and/or symmetric properties of equality. Including the worksheets, there are in excess of 140 problems total, varying in format and difficulty level. A sample of the teacher’s edition of the book section is provided in Figure 1 (Burger, et al., 2007).
Figure 1. Holt 2-3 practice problems and table of associated schemas (Burger, et al., 2007).

In order to practice solving two-step equations with the minimal intrinsic cognitive load, the teacher or student must sort through 140 or more problems to find those 15 that involve only integers and no commutation or symmetry.
Unfortunately, this can be a relatively small number of those problems for students in need of practice. The proposed TI-Nspire program would supplement this book section effectively, facilitating practice by randomly generating focused practice problems. The two-step equation practice portion of the program would have the potential to generate thousands of unique two-step equations, each having integral coefficients and an integral solution.

In analyzing the existing curriculum, the author has focused on equations that are the least cognitively taxing, but a similar argument could be made for any level of complexity in the equations presented in the Holt text. For practice at any particular difficulty level, the quantity of problems offered by the text is both limited and scattered.

Averting cognitive overload.

Deliberate practice can also be used to supplement text-based problem sets that may impose an overly taxing cognitive load for some students. The selected book section requires students to solve two-step equations as well as equations that involve more complex algebraic operations such as distributing and combining like terms. Figure 1 displays a sample of the problem set and summarizes the schema associated with the less-complicated equation solving problems #24-41. The schemas indicated with asterisks are those that are necessary to solve a two-step equation, and if this is the desired skill to practice, the rest of the list becomes a very large intrinsic (possibly even extraneous) load.
algebraic properties of equality*
  o operations
  o commutative
  o symmetric
equivalent expressions
  o distributing
  o combining like terms
inverse operations*
fraction operations
  o add/subtract
  o multiply/divide
  o reciprocals
  o mixed numbers
decimal operations
  o add/subtract
  o multiply/divide
integer operations*
  o add/subtract
  o multiply/divide

Struggling students who have not mastered the associated schema can become overtaxed when working through a problem set with such a large cognitive load. For these students, this cognitive overload creates a gap in the learned curriculum, and that gap can be filled by deliberate practice which automates the related schema. As students achieve success with the less taxing two-step equations, the TI-Nspire programs will present them with more complex equations that require the use of additional schemas. Because the program automatically adjusts difficulty levels based on user performance, students receive focused, deliberate practice, and repetition is limited to manageable amounts. Within a few practice sessions, students will encounter problems similar to #24-41, but the application of the related schema will be introduced much more systematically.

Review of Existing Practice Software

A plethora of computer-based and calculator-based Algebra 1 practice currently exists, and the project’s TI-Nspire programs will add to this body of resources in unique and substantial ways. In the subsequent sections, the author reviews online practice and finds that it is lacking in organization, quantity, and
difficulty level. The author also reviews a stand-alone software package, an Algebra 1 course offered by Carnegie Learning.

**Online practice.**

The author reviewed a variety of websites that offer practice on solving linear equations. From the twenty or more reviewed, four are discussed here in detail. All four websites are free of charge. The website coolmath.com has six levels of equation solving practice available on its Algebra Cruncher's page (Coolmath.com, Inc, 2010), and they are appropriately separated by topic, such as number of steps and difficulty of number concept. User responses are graded for correctness, but performance is not tracked. There are visual examples of the levels that make the page user-friendly as well. The levels, however, are not ordered, and none of them require use of the distributive property or combining like terms. Coolmath.com also features practice with linear functions. It generates problems for students to find slope given two points and also given a linear function in standard form. It also generates problems requiring students produce the standard form of a linear equation given two points and also given a point and a slope. The types of problems for linear functions practice are displayed in a user-friendly way, but they are not organized by difficulty.

Another useful website for developing equation practice is shodor.org, which has an Algebra Quiz page (Shodor, Inc., 2011). The Algebra Quiz generates repetitive practice problems and grades user solutions, but it does not track progress within a single session. This website has some very interesting features, such as an optional and adjustable timer which requires the user to
input an answer before time runs out. It also does a great job of providing the user options of having “all whole numbers” (Shodor, Inc.) in the problems or adjusting the difficulty level of number concept. The site also allows a user to choose whether to include distributing or variables on both sides. The only clearly indicated control of task in the Algebra Quiz page is the option to restrict the problems to whole numbers only.

The website AAAmath.com also provides practice solving linear equations (Branfill, 2009). This website contains lessons and generates practice problems ordered by difficulty level. Solutions are checked, and progress is tracked within each lesson. Several types of time challenge are available to add an element of difficulty to the practice. Unfortunately, the lessons only cover up to two-step equations and never involve distributing, combining like terms, or variables on both sides.

This project’s TI-Nspire programs differ from those found online in several ways. First, each follows a somewhat linear progression through hierarchical topics. Most of the practice websites have various difficulty levels, but only two of the four reviewed put them in order of complexity. The Interactive Algebra site has a progression of 20 problems that can be done for practice, and they gradually introduce cognitive loads in an effective way. The shortcoming of this site is that it only provides one problem of each type. There is no repetitive practice for the various levels. AAAmath.com has four levels that progress nicely and also incorporate repetition, but the difficulty only builds to two-step equations, never covering distributing or combining like terms. Beyond two-step
equations, the author did not find a website that offered repetitive practice along with difficulty levels that showed a clear progression.

Another difference is the level of user guidance required. Most of the websites require the user to adjust difficulty levels manually by changing parameters, or they require the user to decide when to stop a given topic and move on to a more difficult one. In contrast, the TI-Nspire programs will automatically adjust the difficulty level of the problems and inform the user when a new skill level has been reached. They will also notify the user and automatically adjust the difficulty level if easier problems are appropriate. From a classroom instruction standpoint, this control over difficulty level is an important distinction, because it determines whether the teacher or the student is responsible for determining the focus of the deliberate practice. The automation of the difficulty level adjustment makes the programs easier for the teacher and student to use.

Another major drawback of online practice is that it requires a browser and an Internet connection. Unless there is a class set of devices available, the Internet practice can only be done independently outside of class, and can only be done when the student has Internet access.

*Stand-alone software.*

There are also many computer-based curricula that attempt to tutor and/or teach Algebra 1 content. These can be downloaded via the Internet or they are
ordered via mail; in either case, the programs are installed and accessed on the
user’s computer, and many do not require Internet access to function.

One such curriculum is the Algebra 1 course offered by Carnegie
Learning. This curriculum is driven by Cognitive Tutor, which “is based on the
ACT-R theory of learning, memory and performance, which has been validated
by hundreds of lab and field studies” (Carnegie Learning, Inc, 2010). This
complete Algebra 1 curriculum contains both paper and electronic resources, and
the software is intended to be used two days per week. The author reviewed a
lesson on linear models in which the user is presented with the task of creating
variables, a table, and a graph that model a word problem.

The lesson explores multiple representations, having the user create an
expression and eventually plot a linear graph. It focuses on multiple
representations, but the lesson also involves solving linear equations,
substitution, identification of relevant variables, and formulation of expressions.
Based on the complexity of the algebra tasks, the lesson has a relatively high
intrinsic cognitive load. As the user works through the problem, the Cognitive
Tutor gives instantaneous feedback and offers hints when the user enters
incorrect answers. In many cases, the program will often simply give the user the
correct input after several incorrect responses. After the user works a problem,
the tutor generates a new word problem with a different real-world context and a
similar underlying structure. The Cognitive Tutor also tracks progress on distinct
skills such as “calculate input value” and “writing expression” as a user works
through several problems successively.
As direct instruction, the Cognitive Tutor program presents a static website that contains notes and examples. Relevant terms are highlighted in the notes, and definitions are available as hyperlinks. After viewing the lesson, the user encounters a word problem accompanied by questions, an empty table, and an empty graph. The user is expected to identify and define the relevant variables, use them as column headings in a table, and create an expression that describes a linear relationship. The questions prompt the user to evaluate the expression at particular input values and also work backward from the output value to find the input value. After several points are gathered, the user must choose appropriate boundaries for a graph and plot the line on coordinate axes.

Carnegie Learning’s Cognitive Tutor seems like it would be a great supplement to any curriculum and it may have potential as a stand-alone curriculum. It seems to be solidly based in cognitive learning theory, and it offers a level of differentiation that is impressive. Unfortunately, its cost and implementation method are large barriers to classroom implementation.

**Novel and Effective Implementation**

The TI-Nspire provides an effective platform for disseminating deliberate practice, and the particular types of programs proposed for the project are not currently available for the TI-Nspire. The following subsections discuss the availability of TI-Nspire technology in CCISD classrooms and the ways in which this project’s programs would add to the existing resources available for the Nspire calculator.
Availability of Technology

As a result of a 2008 City of Corpus Christi bond initiative, CCISD committed to providing each secondary mathematics classroom with a class set of TI-Nspire handheld technology. Presently, TI-Nspire Navigator technology has been installed at every high school, and TI-Nspire handheld calculators have been provided to every high school (CCISD, 2010). Information about middle school implementation is unclear. Because of its ubiquity in the district’s secondary mathematics classrooms, the TI-Nspire handheld is an effective vector implementing this curricular supplement.

This is particularly important, because access to technology has been identified as a primary factor in effective technology implementation. Regarding the impact of technology, research has shown that three key elements for higher student achievement are the teacher’s attitude, technology competence, and access to tools (Knezek, Christensen, & Fluke, 2003). With regard to access to tools, both teachers and students in CCISD Algebra 1 classrooms have daily access to the TI-Nspire handhelds. Every Algebra 1 teacher in CCISD should have the resources to effectively implement the TI-Nspire programs if they recognize a need for supplementing their existing Algebra 1 curriculum with deliberate practice in linear equations, functions, or systems.

Existing TI-Nspire Documents and Programs

The current body of work on TI-Nspire calculators does not include a similar program to the ones proposed for this project. Texas Instruments (TI), the maker of the Nspire handheld, supports an activities exchange where teachers...
can share lessons developed for the Nspire handheld. A search for Algebra 1 activities on equations and inequalities yielded 37 lessons for the TI-Nspire (Texas Instruments, 2011). The focus of these activities is generally multiple representations and higher-order thinking, and the tasks are highly interactive, implementing TI’s focus on “Action-Consequence” documents (Dick & Burrill, 2010). TI has made a conscious decision to emphasize and promote activities on the TI-Nspire that focus on the latter two levels of the Mathematical Tasks Framework, procedures with connections and complex, non-algorithmic thinking (Dick & Burrill). While their activities certainly hit the mark in this regard, none of the activities on the TI Activities Exchange (TI, 2011) or on the Math Nspired (TI Math Nspired, 2011) websites involve deliberate practice. Teachers must of course help students to develop higher order thinking skills, but TI’s activities would also be supplemented effectively by programs that generate targeted, responsive, deliberate practice.

Justification

The proposed work will address an important topic in an Algebra 1 curriculum, one- and two-variable linear equations. By utilizing CLT approaches and leveraging modern technology, it will implement deliberate practice in a pedagogically sound manner. Further, this proposal demonstrates that existing curricula as well as existing practice software would be well supplemented by this project’s deliberate practice programs. The platform for the deliberate practice is particularly advantageous, because TI-Nspire calculators should be available to every Algebra 1 teacher and student in CCISD. This project should be
considered as a significant component leading to the receipt of a master’s degree in mathematics, because (1) it would add significantly to the curricular supplements available to CCISD teachers, (2) it would do so in a novel and effective manner, (3) and its successful implementation would benefit Algebra 1 students by automating schemas associated with highly relevant skills.
Planned Actions

Beginning in January 2011, the author researched the means for creating the deliberate practice programs and also researching the cognitive principles that would guide such practice. Also in January, the author implemented a test version of the one-variable equation solving program and observed student success. Notably, the class average on a summative equation solving assessment showed high levels of student mastery in both of the author’s Algebra 1 courses. Throughout the spring semester, the author further researched the guiding principles of the project and also developed and implemented more primitive versions of the two-variable equations and the systems of two-variable equations programs. The two latter programs were more limited in scope, and student success was less observable.

The programs will be completed and will be refined through classroom use during Fall 2011. Accompanying documentation will also be created and refined. For linear equations and functions, the TI-Nspire programs along with teacher and student documentation will be prepared in advance of classroom use, and the author will refine them based on classroom usage. The calendar for the 2010-2011 academic year guides the timeline for this process. For linear systems, the content will be covered in the classroom after the project has been defended, so the documents and programs will be created, but they likely will not be refined through classroom usage. Table 4 presents a more detailed timeline describing the project’s completion.
<table>
<thead>
<tr>
<th>Time Period</th>
<th>Planned Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>January 2011</td>
<td>• Begin Research</td>
</tr>
<tr>
<td></td>
<td>• Test version of one-variable equation solving program</td>
</tr>
<tr>
<td>February 2011</td>
<td>• Continue research</td>
</tr>
<tr>
<td></td>
<td>• Test version of two-variable systems of equations program</td>
</tr>
<tr>
<td>March 2011</td>
<td>• Continue research</td>
</tr>
<tr>
<td>April 2011</td>
<td>• Continue research</td>
</tr>
<tr>
<td>May 2011</td>
<td>• Test version of two-variable equation solving program</td>
</tr>
<tr>
<td>June 2011</td>
<td>• Continue research</td>
</tr>
<tr>
<td></td>
<td>• Write and submit proposal</td>
</tr>
<tr>
<td>July 2011</td>
<td>• Continue research</td>
</tr>
<tr>
<td></td>
<td>• Revise proposal</td>
</tr>
<tr>
<td></td>
<td>• Defend proposal</td>
</tr>
<tr>
<td>August 2011</td>
<td>• Complete one-variable equation program</td>
</tr>
<tr>
<td></td>
<td>• Design documentation for one-variable equation program</td>
</tr>
<tr>
<td></td>
<td>• Implement one-variable equation program and documentation</td>
</tr>
<tr>
<td></td>
<td>• Revise program and documentation</td>
</tr>
<tr>
<td>September 2011</td>
<td>• Revise one-variable equation program</td>
</tr>
<tr>
<td></td>
<td>• Complete two-variable equation program</td>
</tr>
<tr>
<td></td>
<td>• Design documentation for two-variable equation program</td>
</tr>
<tr>
<td>October 2011</td>
<td>• Implement two-variable equation program and documentation</td>
</tr>
<tr>
<td></td>
<td>• Revise two-variable equation program and documentation</td>
</tr>
<tr>
<td></td>
<td>• Complete two-variable systems of equations program</td>
</tr>
<tr>
<td></td>
<td>• Design documentation for two-variable systems of equations</td>
</tr>
<tr>
<td>November 2011</td>
<td>• Implement two-variable equation program and documentation</td>
</tr>
<tr>
<td></td>
<td>• Revise two-variable equation program and documentation</td>
</tr>
<tr>
<td></td>
<td>• Revise systems of equations program and documentation</td>
</tr>
<tr>
<td></td>
<td>• Present reading copy of project to committee on November 17th</td>
</tr>
<tr>
<td>December 2011</td>
<td>• Defend Project</td>
</tr>
</tbody>
</table>
End Results Intended

Programs


The TI-Nspire programs will be saved as “.tns” files and stored as data on a CD, and they are intended for teachers to transfer to individual student calculators for usage. The files are typically accessed by uploading them to a TI-Nspire handheld calculator, but the files could also be accessed using TI-Nspire computer software. For informational purposes, the supporting code of the programs will also be presented in a widely available text format.

Documentation

The project will produce documentation for teachers and students using the TI-Nspire programs. The documentation will consist of usage guides for teachers and students, and it will also include worksheets and answer documents as necessary. Supporting documents for teachers and students will be stored as “.pdf” files on a data CD.

Usage

The programs are designed to be introduced after direct instruction on the given topics but before the topics are applied in earnest to open ended problems.
To this end, the worked examples portion of the programs serve well as a review for students, but they are by no means a comprehensive tutor. Likewise, the more open ended demonstration pieces serve as a jumping off point for classroom discussion and should certainly be addressed further by the instructor.

Products Details

A more detailed outline of expected products is outlined below.

TI Nspire Programs

- worked examples
  - solving linear equations, various levels of difficulty
  - manipulating linear functions
- linear equation solving practice
  - randomly generate linear equations
  - various levels of difficulty
  - user ability to choose difficulty level
  - user ability to manually adjust difficulty level
  - automatic level adjustment based on user performance
  - immediate feedback
  - results summary feedback
  - ability to restrict coefficients and solutions to integers
linear function practice:
  - randomly generate 2-variable linear equations
    - standard form
    - slope-intercept form
    - point-slope form
  - ability to restrict coefficients and solutions to integers
  - prompt user to enter relevant information about linear function
    - intercepts
    - equations in various forms
    - points on lines
    - slope

solve systems of equations:
  - randomly generate pairs of linear functions
  - ability to restrict coefficients and solutions to integers
  - prompt user to solve using a particular method
  - prompt user to select a method

Enrichment
  - equation exploration sliders
  - worksheet to accompany equation exploration sliders

Student documentation:
  - instructions
  - prompts
o worksheets
o answer templates

Teacher documentation:

o implementation suggestions
o instructions
o assessment guides
o skill tracker grid
o question prompts/sample answers
o suggested enrichment and accompanying activities
o explanatory document justifying how each TEK is covered by the program.

o summary/description of each program and its functions and levels
Bibliography


