Final Review

College Algebra 1314

Solve the equation by the zero factor property.

1. $10x^2 + 37x + 30 = 0$
   a) $\left\{ \frac{2}{5}, \frac{6}{5} \right\}$
   b) $\left\{ \frac{6}{2}, \frac{5}{2} \right\}$
   c) $\left\{ -\frac{5}{2}, -\frac{6}{5} \right\}$
   d) $\left\{ -\frac{5}{2}, -\frac{6}{5} \right\}$

Solve the equation by square root property.

2. $(3x + 7)^2 = 36$
   a) $\left\{ \frac{29}{3} \right\}$
   b) $\left\{ \frac{1}{3}, \frac{13}{3} \right\}$
   c) $\left\{ -\frac{1}{3}, 0 \right\}$
   d) $\left\{ -\frac{1}{3}, -\frac{13}{3} \right\}$

Solve by completing the square.

3. $x^2 + 3x - 9 = 0$
4. Let \( x = \text{length of short leg} \), \( x + 700 = \text{length of long leg} \), and \( x + 700 + 100 \) or \( x + 800 \) = length of hypotenuse.

\[ (x + 700) + 100 = x + 800 \]

Apply the Pythagorean theorem. Solve for the length of three sides.

a) 300, 1000, 1100
b) 500, 1200, 1300
c) -300, 400, 500
d) 100, 800, 900
e) None of the above

5. The sum of the squares of two consecutive odd numbers is 202. Find the integers.

a) -11 and 9
b) 100 and 102
c) -11 and 9 or 9 and 11
d) 9 and 11
e) None of the above

6. What is the length of the side of a square if its area and perimeter are equal?

a) 2
b) 4
c) 6
d) 8
e) None of the above
7. What is the sum of the solutions for \( \frac{x}{x-2} + \frac{2}{x+2} = \frac{8}{x^2 - 4} \)

a) -6  
b) -4  
c) 2  
d) None of the above

8. A water tank can be filled in 7 minutes and emptied in 8 minutes. If the drain is accidentally left open when the tank is being filled, how long does it take to fill the tank?

a) 15 minutes  
b) 1 minute  
c) 35 minutes  
d) None of the above

9. What is the sum of the solutions for \( \sqrt{x} + 7 + 5 = x \)

(a) 11  
(b) 9  
(c) 2  
(d) None of the above

10. What is the sum of the solutions for \( (x - 1)^4 - 5(x - 1)^2 + 4 = 0 \)

(a) 5  
(b) -3  
(c) -4  
(d) None of the above

Solve the quadratic inequality. Write the solution set in interval notation.

11. \( x^2 + 6x + 8 \geq 0 \)

a) \( (-\infty, -4] \)  
b) \( (-\infty, -4] \cup [-2, \infty) \)  
c) \( [-2, \infty) \)  
d) \( [-4, -2] \)

12. \( \frac{2x + 7}{x - 2} \leq 0 \)
13. Solve the equation.

\[ \frac{3x - 4}{8x + 5} = 3 \]

\[ a) \left\{ -\frac{19}{21}, -\frac{11}{27} \right\} \]

\[ b) \left\{ \frac{10}{27}, -\frac{10}{21} \right\} \]

\[ c) \left\{ -\frac{7}{3}, -\frac{1}{3} \right\} \]

\[ d) \left\{ -\frac{11}{21}, -\frac{19}{27} \right\} \]

Solve the inequality. Write the solution set in interval notation

14. \(|7 - 9x| > 2\)

\[ a) (-1, \frac{5}{9}) \]

\[ b) \left( -\frac{5}{9}, -1 \right) \]

\[ c) \left( -\infty, -\frac{7}{9} \right) \cup \left( -\frac{11}{9}, \infty \right) \]

\[ d) \left( -\infty, \frac{5}{9} \right) \cup \left( 1, \infty \right) \]

15. \(|5x - 4| \leq -1\)

\[ a) \emptyset \]

\[ b) \left( -\frac{4}{5}, \frac{12}{5} \right) \]

\[ c) \left( -\infty, -\frac{4}{5} \right) \]

\[ d) \left( -\infty, -\frac{4}{5} \right) \cup \left( \frac{12}{5}, \infty \right) \]
Find the center and radius of the circle.

16. \( x^2 + y^2 - 8x - 10y + 16 = 0 \)
   a) center: \((4,5)\); radius: 5
   b) center: \((-4,-5)\); radius: 25
   c) center: \((-5,-4)\); radius: 25
   d) center: \((5,4)\); radius: 5

Find the center-radius form of the circle described or graphed.

17. A circle having a diameter with endpoints \((-1,-4)\) and \((-1,4)\)
   a) \((x+4)^2 + y^2 = 1\)
   b) \((x+1)^2 + y^2 = 4\)
   c) \(x^2 + (y+4)^2 = 1\)
   d) \((x+1)^2 + y^2 = 16\)

18. Use the graph to determine the equation of the circle in center-radius form.

   \[ (x - 3)^2 + (y + 4)^2 = 16 \]
   \[ (x + 3)^2 + (y - 4)^2 = 4 \]
   \[ (x - 3)^2 + (y + 4)^2 = 4 \]
   \[ (x + 3)^2 + (y - 4)^2 = 16 \]
19. Give the domain and range of the relation.

\[ f(x) = 4x^2 - 2x - 1 \]

- a) domain \((-\infty, 3]\); range \([0,3]\)
- b) domain \([-3,0]\); range \([-1,3]\)
- c) domain \([0,3]\); range \((-\infty,3]\)
- d) domain \([-1,3]\); range \([-3,0]\)

20. Find \( f(k-1) \) when \( f(x) = 4x^2 - 2x - 1 \)
- a) \(-10k^2 + 4k + 5\)
- b) \(-4k^2 - 6k + 1\)
- c) \(4k^2 - 10k + 1\)
- d) \(4k^2 - 10k + 5\)

21. Find \( f(0) \) if \( f = \{(−2,3),(3,0),(0,5),(5,2)\} \)
- a) 3
- b) 5
- c) (5,3)
- d) None of these
22. Determine the intervals of the domain for which function is increasing, decreasing, and constant.

![Graph of a downward parabola]

a) Increasing $(-\infty,3]$; Decreasing $[3,\infty)$; Constant $[3,\infty)$
b) Increasing $[3,\infty)$; Decreasing $[-\infty,3]$; Constant $(-3,\infty)$
c) Increasing $[3,\infty)$; Decreasing $(-\infty,3]$; Constant $[3,\infty)$
d) Increasing $(-\infty,3]$; Decreasing $(-\infty,3]$; Constant $(-3,\infty)$

Match the description with the correct symbolic expression.

23. A linear equation whose graph has x-intercept -6 and y-intercept 6
   a) $x = 6$
   b) $f(x) = -6$
   c) $y = 6x + 36$
   d) $-6y + 6x = 36$

Find the slope of the line satisfying the given conditions.

24. Through $(7,-9)$ and $(-7,9)$
   a) $\frac{9}{7}$
   b) $\frac{7}{9}$
   c) $\frac{-9}{7}$
   d) $\frac{-7}{9}$
Find the slope of the line and sketch the graph

25. $2x - 3y = -8$  
   A) $m = \frac{3}{2}$  
   B) $m = -\frac{2}{3}$  
   C) $m = \frac{2}{3}$  
   D) $m = -\frac{3}{2}$

26. Find $k$ so that the line through $(-2, 4)$ and $(k, b)$ is
   (a). parallel to $3y - 4x = 1$
   (b). perpendicular to $4y + 5x = 7$

27. When the Celsius temperature is $0^\circ$, the corresponding Fahrenheit temperature is $32^\circ$.
   When the Celsius temperature is $100^\circ$, the corresponding Fahrenheit temperature is $212^\circ$.
   Let $C$ represent the Celsius temperature and $F$ the Fahrenheit temperature.
   (a). Express $F$ as an exact linear function of $C$
   (b). Solve the equation in part (a) for $C$, thus expressing $C$ as a function of $F$.
   (c). For what temperature is $F = C$ a true statement?
28. The table represents a linear function \( f \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-11</td>
</tr>
<tr>
<td>-1</td>
<td>-8</td>
</tr>
<tr>
<td>0</td>
<td>-5</td>
</tr>
<tr>
<td>1</td>
<td>-2</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

(a) Find the slope of the line defined by \( y = f(x) \)
(b) Find the \( y \)-intercept of the line
(c) Find the equation for this line in slope-intercept form

Graph the basic functions:

29. Graph the function \( f(x) = x^2 - 5 \) using the given values of \( x \). Also use the graph to determine the domain and range of the function.

30. For the pair of functions \( f(x) = x^2 \) and \( g(x) = \sqrt{x} \), make the graphs of both the functions in one picture and determine where \( f(x) \geq g(x) \)

31. Graph function \( f(x) = \frac{1}{\sqrt{x+2}} \) and find domain, range for the function.

32. Match the equation, \( y = (x + 2)^2 \), with a description of its graph as it relates to the graph of \( y = x^2 \)
   a) A translation 2 units down
   b) A translation 2 units to the right
   c) A translation 2 units up
   d) A translation 2 units to the left
   e) None of the above
33. Match the equation, \( y = \sqrt{x+3} \), with the sketch of its graph.

A. \[ y \]

B. \[ y \]

C. \[ y \]

D. \[ y \]

E) None of the above
34. Factor \( f(x) = 6x^3 + 19x^2 + 2x - 3 \) into linear factors if -3 is a zero of \( f \).
   a) \( f(x) = (x - 3)(2x + 1)(3x - 1) \)
   b) \( f(x) = (x + 3)(2x - 1)(3x + 1) \)
   c) \( f(x) = (x - 3)(2x + 1)(3x + 1) \)
   d) \( f(x) = (x + 3)(2x + 1)(3x - 1) \)
   e) None of the above

35. Use the factor theorem and synthetic division to decide whether \( x + 2 \) is a factor of 
   \( f(x) = 5x^2 - 14x + 10 \).
   a) No, \( x + 2 \) is not a factor.
   b) Yes, \( x + 2 \) is a factor
   c) Not enough information to decide

36. Find a polynomial function \( f(x) \) of degree 3 with real coefficients that satisfies the given 
   conditions. 
   Zeros of 1, -1 and 0; \( f(2) = 3 \).
   a) \( f(x) = \frac{1}{2} x^2 - 3 \)
   b) \( f(x) = 2x^3 + x - 1 \)
   c) \( f(x) = \frac{2}{3} x^3 + x^2 \)
   d) \( f(x) = \frac{1}{2} x^3 - \frac{1}{2} x \)
   e) None of the above

37. Solve the problem.

   The table shows the population of a city over the past five years.

<table>
<thead>
<tr>
<th>Year</th>
<th>Population (in millions of people)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007</td>
<td>65</td>
</tr>
<tr>
<td>2008</td>
<td>65.5</td>
</tr>
<tr>
<td>2009</td>
<td>67</td>
</tr>
<tr>
<td>2010</td>
<td>69</td>
</tr>
<tr>
<td>2011</td>
<td>71.5</td>
</tr>
</tbody>
</table>
We used these data to develop the quadratic function \( f(x) = 0.037214x^2 + 0.364x + 65 \), which models the population of the city \( y \) in millions in the year \( x \), where \( x = 0 \) represents 2007. Use the model to find the estimated population in 2010.

A) 71,628,000  
B) 66,427,000  
C) 1,490,000,000  
D) 1,490,000

38. Use synthetic division to divide \( (x^3 - x^2 + 3) \), by \( (x + 2) \)
   a) \( x^2 - 3x + 6 - \frac{9}{x+2} \)
   b) \( x^2 - 3x - \frac{3}{x+2} \)
   c) \( x^2 - 3x - 6 + \frac{15}{x+2} \)
   d) None of the above

39. Express \( f(x) = 3x^3 - x^2 + 2x + 5 \) in the form \( f(x) = (x - k)q(x) + r \), when \( k = -1 \)
   a) \( (x - 1)(3x^2 + 2x + 4) + 9 \)
   b) \( (x + 1)(3x^2 - 4x + 6) - 1 \)
   c) \( (x + 1)(3x^2 + 2x) + 5 \)
   d) None of the above

40. Determine if \( k = -3 \) is a solution to \( f(x) = -6x^2 + 3x^2 + x - 186 \). If \( k \) is not a solution give the remainder.
   a) The remainder is 6
   b) The remainder is -324
   c) The remainder is -54
   d) \( k \) is a solution

41. Solve each function.
   a) \( 3^{3x+1}, 9^{3x-1} = 27^3 \)
   b) \( 6^{-x} \cdot 6^{-2x} = \frac{1}{216} \)
   c) \( \frac{81^{3n+2}}{243^{-n}} = 3^4 \)

Solve for \( x \)

42. \( \log(x+7) 11 = 1 \)
   a) \(-4\)
   b) \(\{11\}\)
   c) \(\{18\}\)
   d) \(\{4\}\)
Use the properties of logarithms to rewrite the expression. Simplify the result if possible. Assume all variables represent positive real numbers.

43. \( \log_8 \left( \frac{4\sqrt{2}}{6} \right) \)
   a) \( \log_8 4 + \sqrt{\log_8 5} - \log_8 6 \)
   b) \( (\log_8 4) \left( \frac{1}{2} \log_8 5 \right) - \log_8 6 \)
   c) \( \frac{\log_8 4 + \frac{1}{2} \log_8 5}{\log_8 6} \)
   d) \( \log_8 4 + \frac{1}{2} \log_8 5 - \log_8 6 \)

44. How long will it take for $2100 to grow to $42,600 at an interest rate of 3.2% if the interest is compounded quarterly? Round the number of years to the nearest hundredth.
   a) 377.74 yr
   b) 94.44 yr
   c) 95.56 yr
   d) 636.46 yr

Use the change of base rule to find the logarithm to four decimal places.

45. \( \log_{2.8} 294 \)
   a) 5.9482
   b) 0.1681
   c) 113.0769
   d) 2.4683

Find the value. Give an approximation to four decimal places.

46. \( \ln 287 \)
   a) 0.1762
   b) 5.6595
   c) 105.9041
   d) 2.4579

47. \( \log 2.58 \)
   a) 0.4116
   b) 0.4281
   c) 0.9478
   d) 0.3945

Solve the equation. Round to the nearest thousandth.

48. \( 4^{3x} = 5^{x+1} \)
a) $\{2.161\}$  
b) $\{-7.213\}$  
c) $\{0.631\}$  
d) $\{1.161\}$

Solve the equation and express the solution in exact form.

49. $\log (3 + x) - \log (x - 4) = \log 4$
   a) $\emptyset$
   b) $\left\{ \frac{19}{3} \right\}$
   c) $\left\{ \frac{5}{2} \right\}$
   d) $\left\{ \frac{-19}{3} \right\}$

50. $\ln x - \ln (x - 5) = \ln 9$
   a) $\{4\}$
   b) $\left\{ \frac{45}{8} \right\}$
   c) $\left\{ \frac{5 \ln 9}{\ln 9 - 1} \right\}$
   d) $\emptyset$

51. The growth in population of a city can be seen using the formula $p(t) = 10,696e^{0.008t}$, where $t$ is the number of years. According to this formula, how many years will it take the population to double its year 0 value? Round to the nearest tenth of a year.
   a) 173.2 yr
   b) 65.0 yr
   c) 86.6 yr
   d) 43.3 yr

Find the cofactor $a_{12}$ of the indicated element.

52.
\[
\begin{bmatrix}
-2 & -1 & 0 \\
1 & 1 & 5 \\
3 & 1 & 5
\end{bmatrix}
\]
   a) -10
   b) -24
   c) -2
   d) 10

53. A certain radioactive isotope has a half-life of approximately 1350 years. How many years to the nearest year would be required for a given amount of this isotope to decay to 80% of that amount?
   a) 355 yr
   b) 3135 yr
   c) 435 yr
d) 270 yr

Find the matrix product when possible.

\[
\begin{bmatrix}
3 & -2 \\
5 & 0
\end{bmatrix}
\begin{bmatrix}
0 & -2 \\
3 & 6
\end{bmatrix}
\]

a) \[
\begin{bmatrix}
-6 & -18 \\
10 & 0
\end{bmatrix}
\]

b) \[
\begin{bmatrix}
-10 & 0 \\
39 & -6
\end{bmatrix}
\]

c) \[
\begin{bmatrix}
-18 & -6 \\
-10 & 0
\end{bmatrix}
\]

d) \[
\begin{bmatrix}
-6 & -18 \\
10 & 0
\end{bmatrix}
\]

Find the inverse, if it exists, for the matrix.

\[
\begin{bmatrix}
2 & 4 \\
-1 & 0
\end{bmatrix}
\]

a) \[
\begin{bmatrix}
1/2 & -1 \\
1/4 & 0
\end{bmatrix}
\]

b) \[
\begin{bmatrix}
0 & 1 \\
1/4 & 1/2
\end{bmatrix}
\]

c) \[
\begin{bmatrix}
0 & 1 \\
1/4 & 1/2
\end{bmatrix}
\]

d) \[
\begin{bmatrix}
1/4 & 1/2 \\
0 & -1
\end{bmatrix}
\]

56. Suppose the number of Quickie hamburgers (in millions) served yearly can be modeled by

\[f(x) = 3.54e^{0.15x}\]  Approximate how many years it takes for the number of hamburgers served to reach 1 million. Round to the nearest year.

a) 9 yr
b) 10 yr
c) 8 yr
d) 7 yr
e) 7 yr

57. A certain radioactive isotope has a half-life of approximately 1350 years. How many years to the nearest year would be required for a given amount of this isotope to decay to 80% of that amount?

a) 355 yr
b) 3135 yr
c) 435 yr
d) 270 yr

Use the given row transformation to change the matrix as indicated.

\[
\begin{bmatrix}
3 & 5 & 6 \\
2 & 5 & 1 \\
4 & -5 & 3 \\
\end{bmatrix}
\]
-2 times row 2 added to row 3

\[
\begin{bmatrix}
3 & 5 & 6 \\
2 & 5 & 1 \\
8 & 5 & 5 \\
\end{bmatrix}
\]

a) \[
\begin{bmatrix}
-1 & -5 & 4 \\
2 & 5 & 1 \\
4 & -5 & 3 \\
\end{bmatrix}
\]

b) \[
\begin{bmatrix}
3 & 5 & 6 \\
2 & 5 & 1 \\
0 & -10 & 2 \\
\end{bmatrix}
\]

c) \[
\begin{bmatrix}
3 & 5 & 6 \\
2 & 5 & 1 \\
0 & -15 & 1 \\
\end{bmatrix}
\]

59. John has a jarful of quarters and nickels. There are 88 coins in the jar. The value of the coins is $13.80. How many of each type of coin are there?

a) 41 quarters; 47 nickels
b) 47 quarters; 41 nickels
c) 52 quarters; 36 nickels
d) 83 quarters; 5 nickels

Give the equations of any horizontal asymptotes.

60. \( h(x) = \frac{9x^2 - 9x - 3}{5x^2 - 7x + 6} \)

a) \( y = \frac{9}{5} \)
b) \( y = \frac{9}{7} \)
c) \( y = 0 \)
d) none

Give the equations of any vertical asymptotes.

61. \( h(x) = \frac{5x - 1}{x^2 + 3x - 18} \)

a) \( y = 5 \)
b) \( y = 3, y = -6 \)
c) \( x = 3, x = -6 \)
d) \( x = 3, x = -6 \)

62. Find a polynomial \( f(x) \) of least degree having the graph shown

![Graph of a polynomial showing points at (0, 6), (-2, 0), and (1, 0).]

\[ a) \quad f(x) = (x + 2)(x - 1)^3 \]
\[ b) \quad f(x) = 3(x + 2)(x - 1)^3 \]
\[ c) \quad f(x) = -3(x + 2)(x - 1)^3 \]
\[ d) \quad f(x) = -3(x + 2)(x - 1) \]

Use the graph to answer the question.

63. Find the horizontal and vertical asymptotes of the rational function graphed below.

![Graph of a rational function showing vertical asymptotes at x = -4 and x = 4, and horizontal asymptote at y = 2.]

\[ a) \quad \text{Horizontal: } y = 0; \text{ vertical: } x = \pm 4 \]
\[ b) \quad \text{Horizontal: } y = 2 \quad x = \pm 4 \]
c) Horizontal: none; \( x = \pm 4 \)

d) Horizontal: \( y = \pm 4 \); vertical: \( x = 2 \)

64. Find the domain and range of the rational function graphed below.

\[
\begin{align*}
\text{a) Domain: } (-\infty, -1) \cup (-1, \infty); \text{ Range (1, } \infty) \\
\text{b) Domain: } (-1, \infty); \text{ Range (0, } \infty) \\
\text{c) Domain: } (-\infty, -1) \cup (-1, \infty); \text{ Range (-}\infty, \infty) \\
\text{d) Domain: } (-\infty, -1) \cup (-1, \infty); \text{ Range (0, } \infty) \\
\end{align*}
\]

Decide whether or not the functions are inverses of each other. Use the definition of inverses to determine whether \( f \) and \( g \) are inverses.

\[
f(x) = 3x - 3,
\]
\[
g(x) = \frac{1}{3} x + 1
\]

a) Yes

b) No

If the function is one-to-one, find its inverse. If not, write “not one-to-one.”

65. \( f(x) = x^3 - 3 \)

\begin{align*}
\text{a) } f^{-1}(x) &= \sqrt[3]{x + 3} \\
\text{b) } f^{-1}(x) &= \frac{1}{3} x - 3 \\
\text{c) } f^{-1}(x) &= \frac{1}{3} x + 3 \\
\text{d) Not one-to-one}
\end{align*}

66. \( f(x) = \sqrt[3]{x^2 - 25}, \ x \geq 5 \)

\begin{align*}
\text{a) } f^{-1}(x) &= \sqrt[3]{25 - x^2}, x \leq 0 \\
\text{b) } f^{-1}(x) &= \sqrt{25 + x^2}, x \leq 0 \\
\text{c) } f^{-1}(x) &= \sqrt{25 + x^2}, x \geq 0
\end{align*}
d) \( f^{-1}(x) = \sqrt{25 - x^2}, x \leq 5 \)

Solve the system. If the system has infinitely many solutions, write the solution set with \( x \) arbitrary.

68. \( 6x + 5y = 7 \)
    \( 12x + 10y = 5 \)

a) \( \left\{ x, \frac{4}{5} - \frac{6}{5}x \right\} \)
b) \( \{(0,0)\} \)
c) \( \emptyset \)
d) \( \left\{ x, \frac{7}{5} - \frac{6}{5}x \right\} \)

69. Find the equation of the line \( y = ax + b \) that passes through the points \((-4,4)\) and \((3,-9)\)

a) \( y = -\frac{5}{7}x + \frac{7}{40} \)
b) \( y = -\frac{13}{7}x - \frac{24}{7} \)
c) \( y = -\frac{13}{7}x + \frac{48}{7} \)
d) \( y = -\frac{24}{7}x - \frac{13}{7} \)