Show that the following sequences are decreasing.

1. \( \left\{ \frac{1}{n^2} \right\} \)

2. \( \left\{ \frac{e^{-n}}{n} \right\} \)

3. \( \left\{ \frac{\ln(n)}{n^4} \right\} \) After which term is this sequence decreasing?

Show that the following sequences are increasing and find an upper bound.

4. \( \left\{ \frac{n}{n+4} \right\} \)

5. \( \left\{ 1 - \left( \frac{1}{2} \right)^n \right\} \)

6. \( \left\{ \frac{n^2}{2n^2 + 1} \right\} \)

7. The zombie apocalypse has finally occurred and the government estimates that the spread of the disease will have an infection rate of 0.1. Assuming that the population of the United States is about 300,000,000 the population of healthy people as a function of time in days is \( P_{\text{healthy}} = 300,000,000e^{-0.1t} \) and the population of those either a zombie or dead is \( P_{\text{infected}} = 300,000,000(1 - e^{-0.1t}) \).

a) Is \( P_{\text{healthy}} \) increasing or decreasing?

b) What is the lower and upper bound of \( P_{\text{healthy}} \)?

c) Is \( P_{\text{infected}} \) increasing or decreasing?

d) What is the lower and upper bound of \( P_{\text{infected}} \)?
Compute the limit of the following sequences

8. \[ \left\{ \frac{2n}{n + 7\sqrt{n}} \right\} \]

9. \[ \left\{ \frac{\ln(n)}{n^2} \right\} \]

10. \[ \left\{ \left(1 + \frac{1}{n}\right)^n \right\} \]

11. \[ \left\{ (n + 5)^{1/n} \right\} \]

12. \[ \left\{ \frac{n^2 + 3n^3 - 4}{3 + 2n + 3n^3} \right\} \]

Which of the following statements are always true and which are sometimes false? If the answer is sometimes false, give a counterexample; that is, an example of a sequence which demonstrates that the statement cannot always be true. If the answer is always true, indicate why by referencing a theorem or an idea in the book.

13. If \( \{a_n\} > 0 \) for all \( n \) and \( \lim_{n \to \infty} a_n = L \), then \( L > 0 \).

14. If \( \{a_n\} \) is bounded, then it converges.

15. If \( \{a_n\} \) is decreasing, then it converges.

16. If \( \{a_n\} \) is decreasing and \( \{a_n\} > 0 \) for all \( n \), then it converges.

17. If \( \{a_n\} \) is bounded, then \( \{a_n/n\} \) converges to 0.

18. If \( \{a_n\} \) converges and \( \{b_n\} \) converges, then \( \{a_n/b_n\} \) converges.