Taylor Series Formula
\[ \sum_{n=0}^{\infty} \frac{f^{(n)}(a)(x-a)^n}{n!} \]

Find the Taylor Series for the following functions at the given value by using the definition and finding the derivatives at the given value.

1. \( f(x) = e^x \) at \( a = e \)
2. \( f(x) = e^{2x} \) at \( a = 2 \)
3. \( f(x) = -\sin(x) \) at \( a = \pi \)
4. \( f(x) = \sin(2x) \) at \( a = \frac{\pi}{4} \)
5. \( f(x) = \frac{1}{x} \) at \( a = 7 \)
6. \( f(x) = \ln(x) \) at \( a = 1 \)

Use Maclaurin Series and term by term integration to find a series representation of the following integrals.

7. \( \int \frac{\sin(x)}{x} \, dx \) starting with \( \sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \ldots \)
8. \( \int e^{-x^2} \, dx \) starting with \( e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots \)
9. \( \int \frac{1}{1 + x^2} \, dx \) starting with \( \frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \ldots \)

Find the Maclaurin Series for the left and right side of the equation and verify that they are the same.

10. \( \frac{d}{dx} e^{x^2} = e^{x^2} 2x \)
11. \( \frac{d}{dx} \cos(x) = -\sin(x) \)
12. \( \frac{d}{dx} \sin(3x) = \cos(3x) 3 \)

13. (a) Use differentiation to find a power series representation for
\[ f(x) = \frac{1}{(1 + x)^2} \]
What is the radius of convergence?
(b) Use part (a) to find a power series for
\[ f(x) = \frac{1}{(1 + x)^3} \]
(c) Use part (b) to find a power series for
\[ f(x) = \frac{x^2}{(1 + x)^3} \]